ஸ்ரீ-ல-ஸ்ரீ காசிவாசி சுவாமிநாத சுவாமிகள் கலைக் கல்லூரி
திருப்பனந்தாள் – 612504

S.K.S.S ARTS COLLEGE, THIRUPPANANDAL - 612504

QUESTION BANK

Title of the Paper

CLASSICAL ALGEBRA AND THEORY OF NUMBERS

Course: IIB.Sc (MATHS)

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CORE COURSE VI
CLASSICAL ALGEBRA AND THEORY OF NUMBERS

Objectives

1. To lay a good foundation for the study of Theory of Equations.
2. To train the students in operative algebra.

Unit I

Relation between roots & coefficients of Polynomial Equations – Symmetric functions – Sum of the rth Powers of the Roots

Unit II

Newtion's theorem on the sum of the power of the roots-Transformations of Equations – Diminshing, Increasing & Multiplying the roots by a constant - Reciprocal equations - To increase or decrease the roots of the equation by a given quantity.

Unit III

Form of the quotient and remainder – Removal of terms – To form of an equation whose roots are any power – Transformation in general – Descart's rule of sign

Unit IV

Inequalities – elementary principles – Geometric & Arithmetic means – Weirstrass inequalities – Cauchy inequality – Applications to Maxima & Minima.

Unit V

Theory of Numbers – Prime & Composite numbers – divisors of a given number N – Euler's Function (N) and its value – The highest Power of a prime P contained in N! – Congruences – Fermat's, Wilson's & Lagrange's Theorems.

Text Book(s)

1. T.K.ManickavasagamPillai& others Algebra Volume I.S.V. Publications – 1985 Revised Edition.
2. T.K. ManickavasagamPillai& others Algebra Volume II, S.V.Publications – 1985 Revised Edition.

Unit I : Chapter 6 Section 11 to 13 of (1)
Unit II : Chapter 6 Section 14 to 17 of (1)
Unit III : Chapter 6 Section 18- 21 & 24 of (1)
Unit IV : Chapter 4 of (2)
Unit V : Chapter 5 of (2)

References :

1. H.S.Hall and S.R. Knight, Higher Algebra, Prentice Hall of India, New Delhi.
2. H.S. Hall and S.R.Knight, Higher Algebra, McMillan and Co., London, 1948.

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UNIT I

CHOOSE THE CORRECT ANSWER :

1. Which one is not a polynomial?
   a) $4x^2 + 2x - 1$
   b) $y + 3/y$
   c) $x^3 - 1$
   d) $y^2 + 5y + 1$

2. The polynomial $Px^2 + qx + rx^4 + 5$ is type of
   a) linear
   b) quadratic
   c) cubic
   d) bi- quadratic

3. The zero polynomial $P(x) = 2x + 5$ is
   a) 2
   b) 5
   c) 2/5
   d) $-5/2$

4. The roots of the quadratic equation is $6x^2 - x - 2 = 0$
   a) $2/3, 1/2$
   b) $-2/3, 1/2$
   c) $2/3, -1/2$
   d) $-2/3, -1/2$

5. If -5 is a roots of the equation $2x^2 + px - 15$ =0, then
   a) 3
   b) 5
   c) 7
   d) 1

6. The roots of the equation $7x^2 + x - 1 = 0$ are
   a) Real and distinct
   b) Real and equal
   c) Not real
   d) None of these

7. The sum and product of the roots of the equation $x^2 - kx + x^2 = 0$ are
   a) k, k^2
   b) k^2, k
   c) $-k, k^2$
   d) $k, -k^2$
8. The sum, and product of the roots of the equation is $4x^2 + 7 - 3 = 0$ are
   a) $-\frac{3}{4}, -\frac{7}{4}$
   b) $-\frac{7}{4}, -\frac{3}{4}$
   c) $-\frac{3}{4}, 0$
   d) None of these

9. If α and β are the roots of $x^2 - 2x + 3 = 0$ then the equation with roots 1/ α, 1/ β is
   a) $x^2 - 6x + 11 = 0$
   b) $x^2 + 6x - 11 = 0$
   c) $x^2 - 11x + 6 = 0$
   d) $3x^2 - 2x + 1 = 0$

10. The number of zeros of $x^2 + 4x + 2$
    a) 1
    b) 2
    c) 3
    d) None of these

ANSWERS: 1) b 2) d 3) d 4) c 5) c 6) a 7) a 8) b 9)d 10)b

2 Marks :

11. Define algebraic equation.
12. Define roots of the equation.
13. If α, β we are the roots of $2x^2 + 3x + 5 = 0$,find $\alpha + \beta$ and $\alpha\beta$.
14. If α, β, γ are the roots of the equation $x^3 - px^2 + qx - r = 0$,find the value of $\sum \alpha^2$
15. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$,find the value of $\sum \alpha^2\beta$
16. If α, β, γ are the roots of the equation $x^3 + 3x^2 - 12x + 7 = 0$,find the value of $\sum \alpha^2$
17. Find any one of the roots of the equation $27x^3 + 42x^2 - 28x - 8 = 0$are in geometric progression.
18. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$,find the value of $\sum \alpha^2$

19. If α, β, γ are the roots of the equation $x^3 + px + r = 0$,find the value of $\frac{1}{\beta+\gamma} + \frac{1}{\gamma+\alpha} + \frac{1}{\alpha+\beta}$
20. Write the symmetric function of the roots.

5 Marks :

21. Solve the equations $81x^3 - 18x^2 - 36x + 8 = 0$ whose roots are in harmonic progression.
22. Find the biquadratic equation $ax^4 + 4bx^3 + 6cx^2 + 4dx + e = 0$ behave two pairs of the equal roots.
23. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$ to prove that $(\alpha + \beta)(\beta + \gamma)(\gamma + \alpha) = r - pq$.
24. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$,find the value of $\sum \alpha^2\beta^2$ and $\sum \alpha^4$
25. Find the sum of the 5th powers of the roots of the equation
$$x^4 - 7x^2 - 4x - 3 = 0$$
26. Solve $x^3 - 12x^2 + 39x - 28 = 0$ if its roots are in A.P
27. Prove that the sum of the 11th powers of the roots of the equation is $x^7 + 5x^4 + 1 = 0$ is zero.
28. Find the sum of the 4th powers of roots of the equation $x^3 - 2x^2 + x - 1 = 0$.
29. Find the sum of the cubes of the roots of the equation $x^5 - x^2 - x - 1 = 0$
30. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^3 + ax^2 + bx + c = 0$ from the equation whose roots are $\alpha\beta,\ \beta\gamma,\ \gamma\alpha$.

10 Marks :

31. Show that the roots of the equation $x^3 + px^2 + qx + r = 0$ are in A.P if $2p^3 - 9pq + 27r = 0$.
32. Find that the conditions that roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$ may be in G.P solve equation $27x^3 + 42x^2 - 28x - 8 = 0$whose roots are in G.P
33. If the sum of the roots of two equations $x^4 + 3x^3 + 2x^2 + 6x + s = 0$ equals the sum of the other two,prove that $p^3 + 8r = 4pq$.
34. Solve the equations$x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$, given that two of its roots are equal magnitude and opposite signs.
35. Solve $x^3 + 3ax^2 + 3bx + c = 0$ are in H.P show that $2b^3 = c(3ab - ca)$
36. Solve the harmonic progression of the equation $6x^3 - 11x^2 - 3x + 2 = 0$.
37. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^3 + px^2 + qx + r = 0$ from the equation whose roots are $\beta + \gamma - 2\alpha,\ \gamma + \alpha - 2\beta, \alpha + \beta - 2\gamma$.
38. Find the sum of the 5th powers of the roots of the equation $x^4 - 3x^3 + 5x^2 - 12x + 4 = 0$
39. Solve the equation $x^4 - 2x^3 - 21x^2 + 22x + 40 = 0$, whose roots are in A.P

40. Find the sum of the 6th powers of the roots of the equation $x^7 - x^4 + 1 = 0$

UNIT II

CHOOSE THE CORRECT ANSWER :

1. When X is replaced by $\frac{1}{x}$ and given equation remains unchanged then it is said to be
   a) Linear equation
   b) Radical equation
   c) Quadratic equation
   d) Reciprocal equation

2. $\frac{1}{\alpha} + \frac{1}{\beta}$ is equal to
   a) $\frac{1}{\alpha}$
   b) $\frac{1}{\beta}$
   c) $\frac{\alpha+\beta}{\alpha\beta}$
   d) $\frac{\alpha-\beta}{\alpha\beta}$

3. If α and β are the roots of $3x^2 + 5x - 2 = 0$ then $\alpha + \beta$ is should be
   a) $\frac{5}{2}$
   b) $-\frac{5}{3}$
   c) $\frac{2}{3}$
   d) $\frac{3}{5}$

4. The quotient when 19 is divided by 6 is
   a) 1
   b) 2
   c) 3
   d) 0

5. The remainder when 111 is divided by 12 is
   a) 0
   b) 1
   c) 2
   d) 3

6. If $b^2 - 4ac < 0$, then roots of $ax^2 + bx + c = 0$ are
   a) equal

b) irrotational
c) rational
d) imaginary

7. The number of methods to solve a quadratic equation are
   a) 2
   b) 3
   c) 4
   d) 5
8. Find the reciprocal of $4/9$
   a) 7/4
   b) 9/5
   c) 9/7
   d) 9/4

9. If one of the root of the equation $4x^2 - 2x + P - 4 = 0$ be the reciprocal of other than value P is
   a) 8
   b) -8
   c) -4
   d) 4

10. If $1/2$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$ then the value of k is
   a) 2
   b) -2
   c) 3
   d) -3

ANSWERS: 1) d 2) c 3) b 4) c 5) d 6) d 7) b 8) d 9) a 10) a

2 Marks :

11. Define transformation of equation.
12. Change the equation $2x^4 - 3x^3 + 3x^2 - x + 2 = 0$ into another coefficient of whose highest term will be unity.
13. Write remove the fractional coefficient from the equation $x^3 - \frac{1}{4}x^2 + \frac{1}{3} - 1 = 0$
14. Define reciprocal equation.
15. Solve the equation $x^4 - 5x^3 + 7x^2 - 4x + 5 = 0$ diminishing by 2.
16. Solve $x^7 + 4x^5 + x^3 - 2x^2 + 7x + 3 = 0$ change by signs of the roots.
17. Define reciprocal roots.
18. Discuss the nature of the roots $x^5 - 6x^2 - 4x + 5 = 0$
19. Discuss the nature of the roots $x^6 + 3x^5 + 5x - 1 = 0$
20. Multiply theroots of $x^3 - 3x + 1 = 0$ by 10

5 Marks:

21. Find the equation whose roots are $x^5 - 4x^4 + 3x^3 - 4x + 6 = 0$ diminish by 3
22. Find the equation whose roots are $x^4 - 5x^3 + 7x^2 - 17x + 11 = 0$ each is diminish by 2.
23. Solve the equation $x^4 + 4x^3 - 2x^2 - 12x - 2 = 0$.By transforming this equation into another whose roots are increased by unity.

24. Find the roots $x^5 + 4x^4 + 3x^3 + 3x^2 + 4x + 1 = 0$.
25. Solve $x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$.
26. Discuss the nature of the roots $x^5 - 6x^2 - 4x + 5 = 0$.
27. Solve the equation $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$.
28. Remove the fractional coefficient from the equation $x^3 + \frac{1}{4}x^2 - \frac{1}{16}x + \frac{1}{72} = 0$.
29. Increase by 7 the roots of the equation $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$.
30. Solve $60x^4 - 736x^3 + 1433x^2 - 736x + 60 = 0$.

10 Marks :

31. a) Show that the equation $x^4 - 3x^3 + 4x^2 - 2x + 1 = 0$ can be transformed into a reciprocal equation by diminishing the roots by unit.
    b) Remove the fractional coefficient from the equation $2x^3 + \frac{3}{2}x^2 - \frac{1}{8}x - \frac{3}{16} = 0$
32. Solve $6x^5 - x^4 - 43x^2 + x - 6 = 0$.
33. Solve $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$
34. Solve the equation $2x^6 - 9x^5 + 10x^4 - 3x^3 + 10x^2 - 9x + 2 = 0$
35. Find the equation whose roots are of the equation $x^4 + 8x^3 + 12x^2 - 16x - 28 = 0$.
36. Solve the equation $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.
37. Solve $x^4 + 3x^3 - 3x - 1 = 0$.
38. Find the equation whose roots are the roots of the equation $4x^5 - 2x^3 + 7x - 3 = 0$each is increased by 2.
39. Find the sum of the 9th powers of the roots of the equation is $x^3 + 3x + 9 = 0$is zero.
40. Find the sum of the 5th powers of the roots of the equation is $x^4 - 7x^2 - 4x - 3 = 0$is zero.

UNIT III

CHOOSE THE CORRECT ANSWER :

1. Find the maximum number of positive and negative real zeroes for the equation $y = x^3 - x^2 - x - 1$
   a) One positive and one negative
   b) One positive and 2 negative
   c) Two positive and one negative
   d) None of these

2. Find the maximum number of positive and negative real zeroes for the equation $y = 2x^3 + x - 3$
   a) No positive and one negative
   b) One positive and no negative
   c) One positive and 2 negative
   d) None of these

3. Find the remainder R when $3x^3 + 8x^2 + 8x + 12$ is divided by $x - 4$

a) 214
b) 264
c) 364
d) 518

4. Find the remainder R when dividing the polynomial $2x^2 + x$ by $x - 1$
a) 3
b) 2
c) 1
d) None of these

5. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2,then the value of k is
a) 10
b) -10
c) 5
d) -5

6. If one zero of the quadratic polynomial $(k-1)x^2 + kx + 1$ is -3, then the value of k is
a) $\frac{4}{3}$
b) $-\frac{4}{3}$
c) $\frac{2}{3}$
d) $-\frac{2}{3}$

7. The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are
a) Both positive
b) Both negative
c) One positive and one negative
d) None of these

8. The solution of quadratic equation $x^2 + 5x - 6 = 0$ is
a) $x = -1, x = 6$
b) $x = 1, x = -6$
c) $x = 1$
d) $x = 6$

9. If $a < 0$, then the function $f(x) = ax^2 + bx + c$ has
a) Maximum value
b) Minimum value

c) Constant
d) None of these

10. If $x^4 - 3x + 5$ is divided by $2x - 1$, then the remainder is
a) $\frac{35}{16}$
b) $-\frac{35}{16}$
c) -9
d) 3

ANSWERS: 1) b 2) b 3) c 4) a 5) b 6) a 7) b 8) b 9) a 10) c

2 Marks :

11. Find the quotient and remainder, when $3x^3 + 8x^2 + 8x + 12 = 0$ is divided by $x - 4$.
12. Find the quotient and remainder, when $2x^6 + 3x^5 - 15x^2 + 2x - 4 = 0$ is divided by $x + 5$.
13. Write Descartes's rule of sign.
14. Define complete equation.
15. Change of sign the equation is $x^7 + 4x^5 + x^3 - 27x^2 + 7x + 3 = 0$
16. Transform the equation $3x^3 + 4x^2 + 5x - 6$ into one in which of the coefficient is x^3 is unity.
17. Find the quotient and remainder, when $x^4 - 5x^3 + 7x^2 - 4x + 5 = 0$ is divided by $x - 2$.
18. Multiply by the roots of the equation is $x^4 + 2x^3 + 4x^2 + 6x + 8 = 0$ by ½.
19. Determine completely the nature of the roots of the equation is

$$x^5 - 6x^2 - 4x + 5 = 0$$

20. If Increase by 7 the roots of the equation $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$

5 Marks :

21. Find the number of the real roots of the equation$x^3 + 18x - 6 = 0$.
22. Discuss the nature of the roots$x^4 + 15x^2 + 7x - 11 = 0$
23. Show that $f(x) = x^4 + 7x^2 + 3x - 5$ has positive and negative and 2 imaginary roots
24. Show that $x^7 - 3x^4 + 2x^3 - 1 = 0$ has atleast imaginary roots
25. Increase by 7 be the roots of the equation $3x^4 + 7x^3 + 5x^2 + \text{x} - 2 = 0$
26. Diminishing the roots of the equation $x^4 - 5x^3 + 7x^2 - 4\text{x} + 5 = 0$ by 2
27. Find the equation whose roots of $4x^5 - 2x^2 + 7\text{x} - 3 = 0$ each is increasing by 2.
28. Find the equation whose roots are the squares of the roots are

$$x^4 + x^3 + 2x^2 + x + 1 = 0$$

29. Find the equation whose roots are the squares of the roots are

$$x^4 + x^3 - 2x^2 + x - 1 = 0$$

30. Prove that the equation $x^4 + 3x - 1 = 0$ has two real and imaginary roots

10 Marks:

31. Show that the equation $x^4 + 5x^3 + 9x^2 + 5x - 1 = 0$ can be transformed into a reciprocal equation by diminishing the roots by 2. Hence solve the equation
32. Find the equation whose roots are the roots of the equation $x^4 + 8x^3 + 12x^2 - 16x - 28 = 0$eachincreased by 2. Hence solve the equation

33. Find the relation between the coefficients in the equation $x^4 + px^3 + qx^2 + rx + s = 0$in order that the coefficients of x^3 and x may be removable by the same transformation
34. Find the numerical value of $(\alpha^2 + 2)(\beta^2 + 2)(\gamma^2 + 2)(\delta^2 + 2)$ where $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 - 7x^3 + 8x^2 - 5x + 10 = 0$
35. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, from the equation whose roots are $\alpha - \frac{1}{\beta\gamma}$ $, \beta - \frac{1}{\gamma\alpha}$ $, \gamma - \frac{1}{\alpha\beta}$
36. If α is a root of $x^2(x+1)^2 - k(k-1)(2x^2 + x + 1) = 0$ prove that $\frac{\alpha+1}{\alpha-1}$ is also a root
37. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, from the equation whose roots are $\alpha^2 + \alpha, \beta^2 + \beta, \gamma^2 + \gamma$
38. Show that $12x^7 - x^4 + 10x^3 - 28 = 0$ has at least four imaginary roots
39. Remove the second term from the equation
    i) $x^3 - 6x^2 + 10x - 3 = 0$
    ii) Solve the equation by removing the second term

$$x^3 - 12x^2 + 48x - 72 = 0$$

40. Find the equation whose roots are the cubes of the roots of $x^4 - x^3 + 2x^2 + 3x + 1 = 0$. If the cube roots of the unity are$1, \omega, \omega^2$ then $(P + Q + R)(P + \omega Q + \omega^2 R)(P + \omega^2 Q + \omega R) = P^3 + Q^3 + R^3 - 3PQR$

UNIT IV

CHOOSE THE CORRECT ANSWER :

1. If $-5 > a$ and $a > b$ then -5 is
    a) Less than a
    b) Greater than b
    c) Greater than a
    d) Less than b

2. By solving the inequality $3(a - 6) < 4 + 9$ is
    a) $a < 9$
    b) $a < 12$
    c) $a < 13$

d) $a < 11$

3. By solving the inequality $\frac{1}{3}(x-3) > \frac{1}{2}(x+2)$ is
   a) $x < -10$
   b) $x < -12$
   c) $x < -14$
   d) $x \leq -15$
4. Given that function $f(x) = 2(x+3)x^2 + x - 2$ has an absolute maximum interval $-2 < x < 1$, the maximum value is
   a) -2
   b) -29
   c) -10
   d) -5

5. For a variable x, then n[th] positive root of the product of $X_1, X_2 \ldots X_n$ are called
   a) Arithmetic mean
   b) Harmonic mean
   c) Standard mean
   d) Geometric mean

6. Find the points of inflection of the function $f(x) = sin2x + x^2$on the interval $0 < x \leq \pi^2$
   a) $0, \pi^4$
   b) $0, \pi^2$
   c) π^6
   d) None of these

7. For function $f(x,y)$ to have minimum value at (a,b) value is ?
   a) $rt - s^2 > 0$ and $r < 0$
   b) $rt - s^2 > 0$ and $r > 0$
   c) $rt - s^2 < 0$ and $r < 0$
   d) None of these

8. For the function $f(x,y)$ to have maximum value at (a,b) value is ?
   a) $rt - s^2 > 0$ and $r < 0$
   b) $rt - s^2 > 0$ and $r > 0$
   c) $rt - s^2 < 0$ and $r < 0$
   d) None of these

9. Discuss minimum value of $f(x,y) = x^2 + y^2 + 6x + 12$
   a) -3
   b) 9
   c) -9

d) 9

10. The point $(0,0)$ in the domain of $f(x,y) = \sin(xy)$ is a point of ________
    a) Saddle
    b) Minima
    c) Maxima
    d) Constant

ANSWERS: 1) b 2) d 3) b 4) a 5) a 6) b 7) b 8) a 9) b 10) d

2 Marks :

11. If a, b, c are positive and not all equal then $(a+b+c)(bc+ca+ab) > 9abc$.
12. Define arithmetic mean.
13. Define geometric mean.
14. Show that $n^n > 1.3.5 \ldots \ldots ..(2n-1)$.
15. State Weirstrass inequality.
16. State Cauchy's inequality.
17. If the perimeter of the triangle is given that the area is greatest when the triangle is equalatered.
18. Prove that ½ $< (\frac{1}{2} . \frac{3}{4} . \ldots .\ 2n-1/2n)^{1/n} < 1$
19. If a, b, x are positive numbers .prove that $1 < \frac{a+x}{b+x} < \frac{a}{b}$ if $a > b$ and $\frac{a}{b} < \frac{a+x}{b+x} < 1$if $a < b$.
20. Show that $(b+c-a)^2 + (c+a-b)^2 + (a+b-c)^2 \geq bc+ca+ab$.

5 Marks :

21. Prove that if $n < 2, (n!)^2 > n^{n.}$

22. If $a_{1}, a_{2}, \ldots \ldots \ldots a_{n},$ be an arithmetical progression. Show that $a_{1,}{}^{2} a_{2,}{}^{2} \ldots \ldots . a_{n,}{}^{2} > a_{1,}{}^{n} a_{2,}{}^{n}$
23. If $x_1 x_2 \ldots \ldots \ldots ..x_n = y^n$ show that $(1+x_1)(1+x_2) \ldots ..(1+x_n) \nless (1+y)^n$
24. If $a_{1}, a_{2}, \ldots \ldots \ldots a_{n},$ are positive and $(n-1)s = a_1 + a_2 + \cdots + a_n$ then prove that$a_{1}, a_{2}, \ldots \ldots \ldots a_{n}, \geq (n-1)^n (s-a_1)(s-a_2) \ldots \ldots .(s-a_n)$
25. Show that if a, b, c are positive unequal quantities then $$ax^{b-c} + bx^{c-a} + cx^{a-b} \nless a+b+c.$$
26. Prove that $8xyz < (y+z)(z+x)(x+y) < 8/3(x^3+y^3+z^3)$.
27. Find the maximum value of $(3-x)^5(2+x)^4$ between -2 and 3.
28. Show that $\frac{b+c}{a} + \frac{c+a}{b} + \frac{a+b}{c} \geq b$
29. Show that $a+b+c\,(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}) \geq 9$
30. If $S = a_1 + a_2 + \cdots a_n$ and show that $$\frac{s}{s-a_1} + \frac{s}{s-a_2} + .. + \frac{s}{s-a_n} > \frac{n^2}{n-1} + \cdots + \frac{s}{s-a_n} > \frac{n^2}{n-1}$$

10 Marks :

31. If a, b, c and α, β ... be all positive, then $(\frac{a\alpha + b\beta + c\gamma + \cdots}{a+b+c+\cdots})^{a+b+c+\cdots} > \alpha^a \beta^b \gamma^c$
32. If x and y are positive quantities whose sum is 4. show that

$$(x + 1/x)^2 + (y + 1/y)^2 \nless 12.1/2$$

33. Show that if $a, b, c \dots \dots .. k$ be n positive quantities

$$\left(\frac{a^2 + b^2 + \cdots k^2}{a + b + \cdots k}\right)^{a+b\dots k} > a^a b^b \dots k^k$$

34. State and prove Weirstrass inequalities.
35. State and prove Cauchy's inequalities.
36. Find the least value of $4x + 3y$ for positive value of x and y subject to the condition $x^3y^2 = 6$.
37. Show that the greatest value of $xyz(d - ax - by - cz)$ is $\frac{d^4}{4^4abc}$ provided that all factors are positive.
38. Find the greatest value of $a^m b^n c^p$ …. When $a + b + c$ …is constant m.n.p…. being positive integers.
39. If $a_1, a_2, \dots . a_n$ are positive and if $a_1, a_2, \dots . a_n$ are all less than 1, then
    (i) $(1 + a_1)(1 + a_2) \dots .. (1 + a_n) < \frac{1}{1-s}$
    (ii) $(1 - a_1)(1 - a_2) \dots .. (1 - a_n) < \frac{1}{1+s}$

40. Show that $(x^m + y^m)^n < (x^n + y^n)^m$ if $m > n$

UNIT V

CHOOSE THE CORRECT ANSWER :

1. The value of 12 mod 3 is
    a) 0
    b) 1
    c) 2
    d) 3

2. The value of 155 mod 9 is
    a) 0
    b) 1
    c) 2
    d) 3

3. If $a \mid b$ and $a \mid c$, then
    a) $a \mid bc$
    b) $c \mid a$
    c) $b \mid a$
    d) $a \mid (b + c)$

4. The quotient and remainder when 18 is divided by 5 is
    a) 2 and 3
    b) 1 and 2
    c) 3 and 2
    d) 3 and 3

5. The number of factors of a prime number are
    a) 2
    b) 3
    c) 1
    d) None of these

6. The number “1” is
    a) Prime number
    b) Composite number
    c) Neither prime number nor composite number
    d) None of these

7. The composite number has
    a) More than 2 factor
    b) infinite
    c) 1 factor
    d) None of these

8. The smallest prime number is
    a) 4
    b) 2
    c) 3
    d) 5

9. The largest composite number less than 40
    a) 31
    b) 37
    c) 33
    d) 39

10. The inverse of 3 modulo 7 is
    a) -1
    b) -2
    c) -3
    d) -4

ANSWERS : 1) a 2) c 3) d 4) d 5) a 6) c 7) a 8) b 9) d 10) b

2 Marks :

11. Define Prime number.
12. Define composite number
13. Write the formula for number and sum of the divisor.
14. Find all the numbers and sum of all divisors.
15. Find the number and sum of all the divisor of 360?

16. Find the smallest number with 18 divisors.
17. Find the highest power of 11, contained in 1000.
18. Show that $n(n+1)(2n+1)$ is divisible by 6.
19. Define congruences.
20. Find the remainder when 2^{1000} is divisible by 17.

5 Marks :

21. Find the remainder obtained in 2^{46} divisible by 47.
22. Show that $(18)! + 1$ is divisible by 437.
23. If P is a prime number ,$(P-1)! + 1$ is divisible by P.
24. Show that $13^{2n+1} + 9^{2n+1}$ is divisible by 22.
25. Show that $x^5 - x$ is divisible by 30.
26. Prove that the 5^{th} power of any integer N has the same units digit as N.
27. If $M = 1.3.5..(P-2)$ where P is an odd prime. show that $M^2 \equiv (-1)^{p+1/2}(mod\ p)$
28. If $a \equiv b\ (modm)$ and $a_1 \equiv b_1 (modm)$ and q,r are integers , then
$$qa + ra_1 = qb + rb_1 (mod\ m)$$
29. If $ax = bx\ (mod\ m)$ and it μ is H.C.F of x,m then $a \equiv b\ (mod\ m/n)$
30. If P is a prime number and $P = 4m + 1$ where m is a positive integer prove that $\{(2m)!\}^2 + 1$ is divisible by P.

10 Marks :

31. State and prove Wilson's theorem.
32. State and prove Lagrange's theorem.
33. Prove the product of r consecutive integers is divisible by r!
34. State and prove Fermat's theorem.
35. Show that the 8^{th} power of any number is of the form $17\ m\ or\ 17m \pm 1$.
36. Show that if x and y are both prime to the prime number n, then $x^{n-1} - y^{n-1}$ is divisible by n, and deduce that $x^{12} - y^{12}$ is divisible by 1365.
37. Show that if n is a prime number and $r < n$,
$$(n-r)!(r-1)! + (-1)^{r-1} = 0(mod\ n)$$
38. Prove that the sum of the integers less than N and prime to it including unity is $\frac{1}{2}N(\phi)(N)$.
39. If $d_1, d_2, \dots d_r$into are of the divisors of N, then show that
$$\phi(d_1) + \phi(d_2) + \cdots + \phi(d_r) = N.$$
40. If x, y, z be three consecutive integers, show that $\left(\sum x\right)^3 - 3\sum x^3$ is divisible by 108.