ஸ்ரீ-ல-ஸ்ரீ காசிவாசி சுவாமிநாத சுவாமிகள் கலைக் கல்லூரி
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QUESTION BANK

Title of the Paper

COMPLEX ANALYSIS

COURSE – III B.Sc., Maths

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CORE COURSE XIII

COMPLEX ANALYSIS

Objectives: To enable the students to

1. Understand the functions of complex variables, continuity and differentiation of complex variable functions, C – R equations of analytic functions.

2. Learn about elementary transformation concepts in complex variable.

3. Know about complex Integral functions with Cauchy's Theorem, power series expansions of Taylor's and Laurant's series.

4. Understand the singularity concepts and residues, solving definite integrals using the residue concepts.

UNIT I :

Functions of a Complex variable –Limits-Theorems on Limits –Continuous functions – Differentiability – Cauchy-Riemann equations – Analytic functions –Harmonic functions.

UNIT II :

Elementary transformations - Bilinear transformations – Cross ratio – fixed points of Bilinear Transformation – Some special bilinear transformations.

UNIT III :

Complex integration - definite integral – Cauchy's Theorem –Cauchy's integral formula –Higher derivatives - .

UNIT IV :

Series expansions – Taylor's series – Laurant's Series – Zeroes of analytic functions – Singularities.

UNIT V :

Residues – Cauchy's Residue Theorem –Evaluation of definite integrals.

TEXT BOOK(S) :

1. S.Arumugam, A.Thangapandi Isaac, & A.Somasundaram, Complex Analysis, New Scitech Publications (India) Pvt Ltd, 2002.

UNIT – I -Chapter 2 section 2.1 to 2.8 of Text Book
UNIT – II -Chapter 3 Sections 3.1 to 3.5 of Text Book
UNIT – III -Chapter 6 sections 6.1 to6.4 of Text Book
UNIT –IV -Chapter 7 Sections 7.1 to 7.4 of Text Book
UNIT – V -Chapter 8 Sections 8.1 to 8.3 of Text Book

REFERENCE(S) :

1. J.N. Sharma, Functions of a Complex variable, Krishna Prakasan Media(P) Ltd, 13th Edition, 1996-97.

2. T.K.Manickavachaagam Pillai, Complex Analysis, S.Viswanathan Publishers Pvt Ltc, 1994.

UNIT – I

CHOOSE THE CORRECT ANSWERS

1. For z = 4+3i the value of $(\text{Re } z)^3$ is
   a) -64
   b) 44
   c) 64
   d) -44
2. If $\bar{z}$ is the complex conjugate of z then
   a) $Re\ z = \frac{z+\bar{z}}{2}$
   b) $Re\ z = \frac{z-\bar{z}}{2}$
   c) $Re\ z = \frac{\bar{z}}{2}$
   d) $Re\ z = \frac{z}{2}$
3. Let $z = x + iy$ then its complex conjugate $\bar{z}$ is
   a) $\bar{z} = xiy$
   b) $\bar{z} = x - iy$
   c) $\bar{z} = x + iy$
   d) $\bar{z} = xy$
4. If $\bar{z}$ is the conjugate of z then
   a) $|\bar{z}| = |z|$
   b) $|\bar{z}| < |z|$
   c) $|\bar{z}| > |z|$
   d) None of these
5. Let $\frac{1}{x+iy}$ then Im z is
   a) $\frac{y}{x+iy}$
   b) $\frac{-y}{x^2+y^2}$
   c) $\frac{y}{x^2+y^2}$
   d) $\frac{-y}{x^2-y^2}$

6. If $\bar{z}$is the complex conjugate of z then

   a) $z\bar{z}=a^2$

   b) $z\bar{z}=|z|^2$

   c) $z\bar{z}=|z|$

   d) none of these

7. If a function f(z) is continuous at z_0 then

   a) f(z) is differentiable at z_0

   b) f(z) is necessarily differentiable at z_0

   c) f(z) is analytic at z_0

   d) none of these

8. The function f(z) is $|z|^2$ differentiable at

   a) $z \neq 0$

   b) No where

   c) z=0

   d) None of these

9. For the function $f(z) = z^2$ the value of derivative at $z = 4$ is

   a) 2

   b) 5

   c) 6

   d) 8

10. If a function $f(z)$ is analytic at a point $z = z_0$ then following statement is false

   a) f is differentiable at z_0

   b) f is defined at z_0

   c) f is not continuous at z_0

   d) f is continuous at z_0.

ANSWERS

1)c 2)a 3)b 4)a 5)b 6)b 7)a 8)c 9)d 10)c

TWO MARK QUESTIONS

11. Find the value of $\lim_{z \to 2} \frac{z^2-4}{z-2}$.

12. Define differentiability.
13. Prove that the function $f(z) = \bar{z}$ is nowhere differentiable.
14. Write the statement of C-R equations.
15. Verify the function f(z)= $|z|^2$ is differentiable or not.
16. Write the statement of complex form of C-R equation.
17. Define C-R equations in polar coordinates.
18. Define analytic functions.
19. Define harmonic functions.
20. Define conjugate harmonic functions.

FIVE MARK QUESTIONS

21. Prove that the function $f(z) = \frac{\bar{\bar{z}}}{z}$ does not have a limit as z→0.
22. Prove that the function $f(z) = z^2$ is differentiable at every point and$f'(z) = 2z$.
23. Prove that the function $f(z) = \sqrt{|xy|}$ is not differentiable.
24. Prove that the function $f(z) = e^x(cosy + isiny)$is differentiable.
25. State and prove complex form of C-R equations.
26. Verify C-R equations for the function $f(z) = z^3$.
27. Prove that $f(z) = zImz$ is differentiable only at $z = 0$.
28. Prove that an analytic function in a region with constant modulus is constant.
29. If $\frac{\partial^2}{\partial x \partial y}$=$\frac{\partial^2}{\partial y \partial x}$ prove that $\frac{\partial^2}{\partial x^2}$+$\frac{\partial^2}{\partial y^2}$=$4\frac{\partial^2}{\partial z \partial \bar{z}}$
30. Prove that $u = 2x - x^3 + 3xy^2$ is harmonic and find its harmonic conjugate also finds the analytic function.

TEN MARK QUESTIONS

31. State and prove C-R equations.
32. State and prove C-R equations in polar co-ordinates.
33. Prove that f(z)=$\begin{cases} \frac{zRez}{|z|} & if\ z \neq 0 \\ 0 & if\ z = 0 \end{cases}$ is continuous at z=0 but not differentiable at z=0.
34. Show that f(z)=$\begin{cases} \frac{xy^2(x+iy)}{x^2+y^4} & if\ z \neq 0 \\ 0 & if\ z = 0 \end{cases}$ is not differentiable at z=0.

35. Prove that any analytic function $f(z) = u + iv$ with $argf(z)$ constant is itself a constant function.

36. Show that $u = log\sqrt{x^2 + y^2}$ is harmonic and determine its conjugate and hence find the corresponding analytic function f(z).

37. Show that $u(x, y) = \sin x \cosh y + 2 \cos x \sinh y + x^2 - y^2 + 4xy$ is harmonic. Find an analytic function f(z) in terms of z with the given u for its real part .

38. Find the analytic function $f(z) = u + iv$ if $u + v = \frac{sin2x}{\cosh 2y - \cos 2x}$

39. Find the analytic function f(z)=u+iv given that $u - v = e^x(cosy - siny)$.

40. If u+v =(x-y)(x^2+4xy+y^2) and f(z)=u+iv find the analytic function f(z)in terms of z.

UNIT-II
CHOOSE THE CORRECT ANSWER

1. The fixed points of the transformation $\omega = \frac{1}{z}$ are
   a) -1&1
   b) 1&2
   c) 1&0
   d) ∞&0
2. The Mobius transformation is
   a) $\omega = \frac{az-b}{cz-d}$
   b) $\omega = \frac{az+b}{cz+d}$
   c) $\omega = az - b$
   d) $\omega = az + b$
3. The cross ratio of (z_1, z_2, z_3, z_4) is
   a) $\frac{(z_1-z_3)(z_2-z_4)}{(z_1-z_4)(z_2-z_3)}$
   b) $\frac{(z_1-z_3)(z_2-z_4)}{(z_1+z_4)(z_2+z_3)}$
   c) $\frac{(z_1+z_3)(z_2+z_4)}{(z_1-z_4)(z_2-z_3)}$
   d) $\frac{(z_1+z_3)(z_2+z_4)}{(z_1+z_4)(z_2+z_3)}$

4. Four distinct points z_1, z_2 , z_3 , z_4 are collinear or concyclic iff (z_1,z_2,z_3,z_4) is
   a) Real
   b) Imaginary
   c) Zero
   d) Infinite
5. A bilinear transformation with only one fixed point is called
   a) Elliptic
   b) Hyperbolic
   c) Parabolic
   d) None of these
6. A bilinear transformation having ∞ as the only fixed point is
   a) Rotation
   b) Translation
   c) Inversion
   d) Magnification
7. A bilinear transformation 0&∞ as the two fixed points are
   a) Translation
   b) Rotation
   c) Inversion
   d) None of these
8. The invariant points of the transformation $\omega = \frac{1}{z-2i}$ is
   a) 1
   b) 2i
   c) 3
   d) i
9. The inverse bilinear transformation of z is
   a) $\frac{-d\omega+b}{c\omega-a}$
   b) $\frac{d\omega+b}{c\omega+a}$
   c) $\frac{-d\omega+b}{c\omega+a}$

d) $\frac{d\omega+b}{c\omega-a}$

10. The transformation w=$\bar{z}$ is
    a) Bilinear transformation
    b) Preserve cross ratio
    c) Not a bilinear transformation
    d) None of these

ANSWERS

1)a 2)b 3)a 4)a 5)c 6)b 7)b 8)d 9)a 10)c

TWO MARK QUESTIONS

11. Define bilinear transformation.

12. Define inverse bilinear transformation.

13. Define cross ratio.

14. Define invariant points.

15. Define hyperbolic.

16. Define elliptic.

17. Define parabolic.

18. Define translation.

19. Find the invariant points of the transformation $\omega = \frac{1+z}{1-z}$

20. Define inversion.

FIVE MARK QUESTIONS

21. Under the transformation $\omega = iz + i$ show that the half plane x>0 maps onto the half plane v>1.

22. Find the image of the square region with vertices (0,0), (2,0), (2,2), (0,2) under the transformation $\omega = (1+i)z + (2+i)$.

23. Show that by means of the inversion $\omega = \frac{1}{z}$ the circle given by $|z-3| = 5$ is mapped into the circle $\left|\omega + \frac{3}{16}\right| = \frac{5}{16}$.

24. Find the image of the circle $|z - 3i| = 3$ under the map $\omega = \frac{1}{z}$

25. Find the image of the strip 2<x<3 under $\omega = \frac{1}{z}$

26. Show that the transformation $\omega = \frac{2z+3}{z-4}$ maps the circle $z\bar{z} - 2(z + \bar{z}) = 0$ into a straight line given by $2(\omega + \bar{\omega}) + 3 = 0$

27. Find the bilinear transformation which maps the points z=-1,1,∞ respectvely on $\omega = -i, -1, i$

28. Find the bilinear transformation which maps the points $z_1 = 0, z_2 = -i, z_3 = -1$ into $\omega_1 = i, \omega_2 = 1, \omega_3 = 0$ respectively

29. Any bilinear transformation having two finite fixed points α & β can be written in the form $\frac{\omega-\alpha}{\omega-\beta} = k\left(\frac{z-\alpha}{z-\beta}\right)$

30. Any bilinear transformation having ∞ and $\alpha \neq \infty$ as fixed points can be written in the form $\omega - \alpha = k(z - \alpha)$

TEN MARK QUESTION

31. Write the short notes on translation, rotation, magnification and inversion.

32. Show that the transformation $\omega = \frac{5-4z}{4z-2}$ maps the unit circle $|z| = 1$ into a circle of a radius unity and centre -1/2.

33. Any bilinear transformation can be expressed as a product of translation, rotation, magnification and inversion.

34. Prove that any bilinear transformation preserves cross ratio.

35. Find the bilinear transformation which maps the points $z_1 = 2, z_2 = i, z_3 = -2$ onto $\omega_1 = 1, \omega_2 = i, \omega_3 = -1$ respectively.

36. Determine the bilinear transformation which maps 0, 1, ∞ into i, -1, -i respectively. under this transformation show that the interior of the unit circle of the z- plane maps onto the half plane left to the v axis (left half of the ω-plane).

37. Find the bilinear transformation which maps -1, 0, 1 of the z- plane onto -1, -i, 1 of the ω-plane. Show that under this transformation the upper half of the z- plane maps onto the interior of the unit circle $|\omega| = 1$.

38. A bilinear transformation $\omega = \frac{az+b}{cz+d}$ where ad-bc$\neq 0$ maps the real axis into itself iff a, b, c, d are real. Further this transformation maps the upper half plane $Imz \geq 0$ into the upper half plane $Im\omega \geq 0$ iff ad-bc>0.

39. Any bilinear transformation which maps the unit circle |z|=1 onto the unit circle $|\omega|$=1 can be written in the form $\omega = e^{i\lambda}\left[\frac{z-\alpha}{\bar{\alpha}z-1}\right]$ where λ is real. Further this transformation maps the circular disc $|z| \leq 1$ onto the circular disc $|\omega| \leq 1$ iff $|\alpha| < 1$.

40. Any bilinear transformation which maps the real axis onto unit circle $|\omega|$=1 can be written in the form $\omega = e^{i\lambda}\left[\frac{z-\alpha}{z-\bar{\alpha}}\right]$ where λ is real. Further this transformation maps the upper half plane Imz≥ 0 onto the unit circular disc $|\omega| \leq 1$ iff Im$\alpha > 0$.

UNIT – III

CHOOSE THE CORRECT ANSWERS

1. The correct statement is

a) $\left|\int_a^b f(t)dt\right| = \int_a^b |f(t)|dt$

b) $\left|\int_a^b f(t)dt\right| \leq \int_a^b |f(t)|dt$

c) $\left|\int_a^b f(t)dt\right| \geq \int_a^b |f(t)|dt$

d) None of these

2. If every simple closed curve lying in D encloses only points of D is

a) Multiply connected

b) Not connected

c) Simply connected

d) None of these

3. The Cauchy's integral formula is

a) $f(z_0) = \frac{1}{2\pi i}\int_c \frac{f(z)dz}{z-z_0}$

b) $f(z_0) = \int_c \frac{f(z)dz}{z-z_0}$

c) $f(z_0) = \frac{1}{2\pi i}\int \frac{f(z)dz}{z-z_0}$

d) None of these

4. The value of $\int_{|z|=1} \frac{e^z}{z} dz$ is

a) 2π

b) 0

c) ∞

d) 2πi

5. Morera's theorem is converse of

a) Liouville's theorem

b) Cauchy's inequality

c) Cauchy's theorem

d) None of these

6. A bounded entire function in the complex plane is

a) Constant

b) zero

c) one

d) infinite

7. The value of $\int_c \frac{dz}{z-a}$ is

a) 1

b) 0

c) 2πi

d) 2π

8. The value of $\int_{|z|=1} \frac{e^z}{z^n} dz$ is

a) $\frac{2\pi i}{(n-1)!}$

b) $\frac{2\pi i}{n!}$

c) $\frac{2\pi i}{(n+1)!}$

d) 0

9. Every polynomial of degree ≥ 1 has at least

a) Two root

b) Three root

c) Four root

d) One root

10. Let c be the curve integration, L the length of c and many constant such that $|f(z)| \leq M$ everywhere on c then the complex line integral is

a) $\left|\int_c f(z)dz\right| \leq ML$

b) $\left|\int_c f(z)dz\right| \leq M/L$

c) $\left|\int_c f(z)dz\right| \leq L/M$

d) None of these

ANSWERS

1)b 2)c 3)a 4)d 5)c 6)a 7)c 8)a 9)d 10)a

TWO MARK QUESTIONS

11. Find the value of $\int_c \frac{1}{z} dz$

12. State the Cauchy's theorem.

13. Define simply connected region.

14. Define multiply connected region.

15. Define Cauchy's integral formula.

16. Define maximum modulus theorem.

17. Find the value of $\frac{1}{2\pi i}\int_c \frac{z^2+5}{z-3} dz$.

18. State the Cauchy's inequality.

19. State the Liouville's theorem.

20. State the fundamental theorem of algebra.

FIVE MARK QUESTIONS

21. State and prove Cauchy's inequality.

22. State and prove Liouville's theorem.

23. State and prove fundamental theorem of algebra.

24. State and prove Morera's theorem.

25. Prove that $\left|\int_a^b f(t)dt\right| \leq \int_a^b |f(t)|\, dt$.

26. Prove that $\left|\int_c f(z)\, dz\right| \leq Ml$ where $M = max\{|f(z)|/z\epsilon C\}$ and l is the length of C.

27. Evaluate $\int_c f(z)dz$ where f(z)=y-x-i3x^2 and C is the line segment from z=0 to z=1+i.

28. Evaluate the integral $\int_c (x^2 - iy^2)dz$ where C is the parabola y=2x^2 from (1, 2) to (2, 8).

29. Evaluate $\int_c \frac{z}{z^2-1} dz$ where C is the positively oriented circle |z|=2.

30. Prove that $\int_c \frac{\sin^2 z}{(z-\pi/6)^3} dz = \pi i$ where C is the circle |z|=1

TEN MARK QUESTIONS

31. Prove that $\int_c \frac{dz}{(z-a)^n} = \begin{cases} 0 & \text{if } n \neq 1 \\ 2\pi i & \text{if } n = 1 \end{cases}$ where C is the circle with centre a and radius r and $n \in z$.

32. Evaluate $\int_c |z|\, \bar{z}\, dz$ where C is the closed curve consisting of the upper semicircle |z|=1 and the segment -1$\leq x \leq 1$.

33. Show that $\int_c |z|^2\, dz = -1 + i$ where C is the square with vertices O(0,0), A(1,0), B(1,1) and C(0,1)

34. State and prove Cauchy's theorem

35. State and prove Cauchy's integral formula.

36. State and prove Maximum modulus theorem.

37. Evaluate $\int_c \frac{e^z}{z^2+4}$ where C is positively oriented circle |z-i|=2

38. Evaluate $\int_c \left(\frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)}\right) dz$ where C is the circle |z|=3

39. Evaluate $\int_c \frac{\sin 2z}{(z-\pi i/4)^4}$ where C is |z|=1

40. Evaluate $\int_c \frac{e^z}{(z+2)(z+1)^2} dz$ where C is |z|=3.

UNIT – IV

CHOOSE THE CORRECT ANSWERS

1. If f(z) is entire function the Taylor's series is
   a) Convergent for all z
   b) Divergent for all z
   c) Constant
   d) None of these
2. An annulus $\rho_1 \leq |z_1 - a| \leq \rho_2$ is
   a) Open set
   b) Closed set
   c) Disconnected set
   d) None of these
3. The value of the derivative of $f(z) = \frac{z-i}{z+i}$ at z=i is
   a) i/2
   b) i/4
   c) –i/2
   d) i/3
4. A function have isolated singularity at
   a) ∞
   b) 0
   c) 1
   d) None of these
5. A function $\frac{1}{z}$ have isolated singularities at
   a) ∞
   b) 0
   c) 1
   d) 2
6. A function $\tan z$ have singularity at
   a) $\left(k + \frac{1}{2}\right)\pi, k = 1,2, \ldots.$

b) $\frac{1}{2}k\pi$

c) $2k\pi$

d) None of these

7. A function f(z)=z^2 have zero of order

a) One

b) Two

c) Three

d) Four

8. A function f(z)=$\cos z$ have zero of order

a) One

b) Two

c) Three

d) Four

9. The types of singularities are

a) One

b) Two

c) Three

d) Four

10. An isolated singularity ‘a’ of f(z) is a pole if $\lim_{z \to a} f(z)$ is

a) 0

b) ∞

c) 1

d) None of these

ANSWERS

1)a 2)b 3)c 4)a 5)b 6)a 7)b 8)a 9)c 10)b

TWO MARK QUESTIONS

11. What is Taylor’s theorem?
12. What is Maclaurin’s series formula?
13. What is Laurent’s theorem?

14. Find the Maclaurin's expansion of log (1+z).
15. Define zero of order.
16. Define singular point.
17. Find the singular point of $\frac{1}{z(z-i)}$
18. What is removable singularity?
19. Define poles.
20. What is an Essential singularity?

FIVE MARK QUESTINS

21. Expand $f(z) = \frac{1}{z}$ into Taylor's series about z=1.
22. Expand $f(z) = \cos z$ into Taylor's series about $z = \frac{\pi}{2}$.
23. Expand ze^{2z} into Taylor's series about z=-1.
24. Find the Taylor's series to represent $\frac{z^2-1}{(z+2)(z+3)}$ in $|z| < 2$.
25. Find the Laurent's series expansion of $f(z) = z^2 e^{1/z}$ about z=0.
26. Find the Laurent's series expansion of $\frac{z}{(z+1)(z+2)}$ about z=-2.
27. Find the singularity of $f(z) = \frac{\sin z}{z}$
28. Find the singularity of $f(z) = \frac{z-\sin z}{z^3}$
29. Find the singularity of $f(z) = e^{1/z}$
30. State and prove Riemann's theorem.

TEN MARK QUESTIONS

31. State and prove Taylor's theorem.
32. Expand $f(z) = \frac{z-1}{z+1}$ as a Taylor's theorem

    (i) About the point z=0

    (ii) About the point z=1

    Determine the region of convergence in each case.
33. Show that

    (i) $\frac{1}{z^2} = 1 + \sum_{n=1}^{\infty}(n+1)(z+1)^n$ when $|z+1| < 1$

(ii) $\frac{1}{z^2} = \frac{1}{4} + \frac{1}{4}\sum_{n=1}^{\infty}(-1)^n(n+1)\left(\frac{z-2}{2}\right)^n$ When $|z-2| < 2$

34. State and prove Laurent's theorem.

35. Expand $\frac{-1}{(z-1)(z-2)}$ as a power series in z in the regions

(i) $|z| < 1$

(ii) $1 < |z| < 2$

(iii) $|z| > 2$

36. Expand $\frac{1}{z(z-1)}$ as Laurent's series

(i) About z=0 in powers of z

(ii) About z=1 in powers of z-1

Also state the region of validity.

37. Expand $f(z) = \frac{z}{(z-1)(2-z)}$ in a Laurent's series valid for

(i) $|z| < 1$

(ii) $1 < |z| < 2$

(iii) $|z| > 2$

(iv) $|z-1| > 1$

(v) $0 < |z-2| < 1$

38. If $f(z) = \frac{z+4}{(z+3)(z-1)^2}$ find Laurent's series expansion in

(i) $0 < |z-1| < 4$

(ii) $|z-1| > 4$

39. Find the Laurent's series expansion of the function $\frac{z^2-1}{(z+2)(z+3)}$ valid in the annular region $2 < |z| < 3$.

40. Determine and classify the singular points of

(i) $f(z) = \frac{z}{e^z-1}$

(ii) $f(z) = \sin\left(\frac{1}{z}\right)$

UNIT – V

CHOOSE THE CORRECT ANSWERS

1. The poles of first order are known as
   a) complex pole
   b) Simple pole
   c) Singularities
   d) None of these
2. If f(z) is analytic and has a pole at $z=z_0$ then
   a) $|f(z)|=C$, as $z \to z_0$
   b) $|f(z)|=0$, as $z \to z_0$
   c) $|f(z)|=\infty$, as $z \to z_0$
   d) None of these
3. The zero of first order is known as
   a) Complex zero
   b) Simple zero
   c) Singularity
   d) Poles
4. The second order zero is
   a) $f(z_0)=f'(z_0)=0 \& f''(z_0)\neq 0$
   b) $f(z_0)=f'(z_0)=0 \& f'(z_0)\neq 0$
   c) $f(z_0)=f'(z_0)=f''(z_0)=0$
   d) None of these
5. If f (z) is entire then
   a) f(z) is analytic for all z
   b) f(z) is diverges for all z
   c) f(z) is not analytic for all z
   d) None of these
6. The residue of $\frac{e^z}{z^2}$ is
   a) 0
   b) 2
   c) 1
   d) ∞
7. The simple poles of $\frac{z+1}{z^2-2z}$ is
   a) Z=1,2
   b) Z=1,1
   c) Z=1,2
   d) Z=0,2
8. The value of $\int_c \frac{\phi'(z)}{\phi(z)}\, dz$ is

a) 0
b) 1
c) ∞
d) 2

9. Cauchy's Residue theorem of $\int_c f(z)dz$ is
   a) 2π Res $\{f(z); z\}$
   b) $2\pi i \sum_{j=1}^{n} \ Res\left\{f(z); z_j\right\}$
   c) Res $\{f(z); z\}$
   d) None of these
10. The value of $\frac{1}{2\pi i}\int_c \frac{f'(z)}{f(Z)}\ dz$ is
   a) N
   b) P
   c) N-P
   d) N+P

ANSWERS

1)b 2)c 3)b 4)a 5)a 6)c 7)d 8)a 9)b 10)c

TWO MARKS QUESTIONS

11. Define Residues.
12. Find the residue of cot z at z=0.
13. Find the residue of $\frac{z+1}{z^2-2z}$ at $z = 0$.
14. State the Cauchy's residue theorem.
15. State the Argument theorem.
16. State the Rouchy's theorem.
17. State the Fundamental theorem of algebra.
18. Evaluate $\int_c \frac{dz}{2z+3}$ where c is $|z| = 2$.
19. Find the residue of $\frac{z+1}{z^2-2z}$ at z=2.
20. Evaluate $\int_c \frac{dz}{z^2e^z}$ where $c = \{z: |z| = 1\}$.

FIVE MARK QUESTIONS

21. Find the residue at z=0 of $\frac{1+e^z}{zcosz+sinz}$.
22. Find the residue of $\frac{1}{(z^2+a^2)^2}$ at z=ai.
23. Find the poles of $f(z) = \frac{z^2+4}{z^3+2z^2+2z}$ and determine the residues at the poles.

24. Use Laurent's series to find the residue of $\frac{e^{2z}}{(z-1)^2}$ at z=1.

25. Evaluate $\int_c \frac{z^2 dz}{(z-2)(z+3)}$ where c is the circle $|z| = 4$.

26. Evaluate $\int_c tanz\, dz$ where c is $|z| = 2$

27. Evaluate $\int_c \frac{e^{2z}}{(z+1)^3} dz$ where c is $|z| = \frac{3}{2}$

28. Evaluate $\int_c \frac{3cosz}{2i-3z} dz$ where c is the unit circle.

29. Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4sin\theta}$ use contour integration.

30. Evaluate $\int_0^{2\pi} \frac{d\theta}{2+cos\theta}$ use contour integration.

TEN MARK QUESTIONS

31. Find the residue of $\frac{e^z}{z^2(z^2+9)}$ at its poles.
32. Find the residue of $\frac{1}{z-sinz}$ at its poles.
33. State and prove Cauchy's residue theorem.
34. State and prove Argument theorem.
35. State and prove Rouche's residue theorem.
36. Prove that $\int_0^{2\pi} \frac{d\theta}{1+asin\theta} = \frac{2\pi}{\sqrt{1-a^2}}$, $-1 < a < 1$.
37. Prove that $\int_0^{\pi} \frac{ad\theta}{a^2+sin^2\theta} = \frac{\pi}{\sqrt{a^2+1}}$, $a > 0$.
38. Use contour integration method to evaluate $\int_0^{\infty} \frac{dx}{1+x^2}$.
39. Prove that $\int_0^{\infty} \frac{dx}{x^6+1} = \frac{\pi}{3}$.
40. Prove that $\int_0^{\infty} \frac{cosx}{1+x^2} dx = \frac{\pi}{2e}$.