ஸ்ரீ-ல-ஸ்ரீ காசிவாசி சுவாமிநாத சுவாமிகள் கலைக் கல்லூரி

திருப்பனந்தாள் – 612504

S.K.S.S ARTS COLLEGE, THIRUPPANANDAL - 612504

QUESTION BANK

Title of the Paper

DIFFERENTIAL EQUATIONS AND LAPLACE TRANSFORMS

COURSE – I B.Sc., Maths

Prepared by

P.ELAKKIYA M.Sc., B.Ed.,
Assistant Professor
Department of Mathematics

DIFFERENTIAL EQUATIONS AND LAPLACE TRANSFORMS

OBJECTIVES:

1. To know the order and degree of the ODE's

2. To identify some specific methods and solve them

3. To make difference between ODE and PDE

4. To solve some standard methods

5. To know the concept of Laplace transforms and it's inverse with applications

UNIT I

First order , higher degree differential equations solvable for x, solvable for y, solvable for dy/dx, Clairauts form – conditions of M dx + N dy = 0 – simple problems.

UNIT II

Particular integrals of second order differential equations with constant coefficients – Linear equations with variable coefficients – Method of variation of parameters (Omit third & higher order equations)

UNIT III

Formation of Partial Differential Equation – General, Particular & Complete integrals – Solution of PDE of the standard forms – Lagrange's method – Solving of Charpit's method and a few standard forms.

UNIT IV

PDE of second order homogeneous equation with Constant coefficients – Particular integrals of the forms e^{ax+by}, $\sin(ax+by)$, $\cos(ax+by)$, $x^r y^s$ and $f(x,y)$.

UNIT V

Laplace Transforms – Standard formulae – Basis theorems & simple applications – Inverse Laplace Transforms – Use of Laplace Transforms in solving ODE with constant coefficients.

TEXTBOOK

1. T.K.Manicavachagom Pillay & S.Narayanan, Differential Equations, S.Viswanathan Publishers Pvt.Ltd. 1996.

2. Arumugam & Isaac, Differential Equations, New Gamma Publishing House, Palayamkottai, 2003.

REFERENCE BOOK:

1 .M.D.Raisinghania, ordinary and Partial Equations, S.Chand & co

2. M.K.Venkatraman, Engineering Mathematics, S.V.Publications, 1985 Revised Edition.

UNIT – I

CHOOSE THE CORRECT ANSWER

1. The degree of the differential equation $\left(1+\frac{dy}{dx}\right)^3 = \left(\frac{d^2y}{dx^2}\right)^2$ is

   A)1
   B) 2
   C) 3
   D) 4

2. The degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log x$ is

   A) 4

   B) 0

   C) 3

   D) Not defined

3. The order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^2 = \frac{d^2y}{dx^2}$ are

   A) 1, 2

   B) 2, 2

   C) 2, 1

   D) 4, 2

4. Solution of differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$ is

   A) $\frac{1}{x} + \frac{1}{y} = c$

   B) $\log x \log y = c$

   C) xy = c

   D) x+y= c

5. The solution of the equation $P^2 - 5P + 6 = 0$ is

   A) 3, 2

   B) 5, 4

   C) 2, 4

   D) 3, 1

6. The value of P is $x + x\mathrm{p}^2 - 2yp = 0$

A) xy

B) x+y

C) $\frac{x+y}{c}$

D) xc

7. $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$ is

A) Not defined
B) Exact
C) Not exact
D) None of these

8. The value of x is $y^2 = 1 + p^2$

A) $\sinh^{-1}\left(\sqrt{y^2 - 1}\right)$+ c

B) $\sinh^{-1}\left(\sqrt{y^2 + 1}\right) +$ c

C) x+y

D) xc - y

9. The solution of P is $Y = 2PX + P^4X^2$

A) xy

B) x+y-2y

C) $\sqrt{\frac{c}{x}}$

D) $\sqrt{\frac{x}{c}}$

10. The value of Y is $x\frac{dy}{dx} = y + \log\frac{dy}{dx}$

A) $x\log p + y$

B) $xp - \log p$

C) $x\log p + p$ (D) $xp + \log P$

ANSWERS:

1. B 2.D 3.C 4.C 5.A 6.D 7.C 8.A 9.C 10.

TWO MARK QUESTIONS

11. $P^2 - 5P + 6 = 0$ To find the values of P.
12. Show that $(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0$ is exact.
13. Solve$y = (\mathrm{x} - \mathrm{a})\mathrm{p} - \mathrm{p}^2$.
14. Solve$a(\mathrm{xdy} + 2\mathrm{ydx}) = \mathrm{xydy}$.
15. To check exact (or) not exact$(\mathrm{a}^2 - 2\mathrm{xy} - \mathrm{y}^2)\mathrm{dx} - (\mathrm{x} + \mathrm{y})^2\mathrm{dy} = 0$.
16. What is differential equation of first order?
17. Define clairaut's form.
18. Write down the condition of integrability of Mdx+Ndy=0.
19. Define integrating factor.
20. Solve$P^2 - 7P + 12 = 0$.

FIVE MARK QUESTIONS

21. Solve $\mathrm{YP}^2 + 2\mathrm{XP} - \mathrm{Y} = 0$.
22. Solve $\mathrm{y}^2\mathrm{dx} - (1 - \mathrm{xy})\mathrm{dy} = 0$.
23. Solve $\left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)^2 - 6\left(\frac{\mathrm{dy}}{\mathrm{dx}}\right) + 8 = 0$.
24. Solve $\mathrm{y}^2 = 1 + \mathrm{P}^2$.
25. Solve $2\left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)^2 + \mathrm{xy}^2 = (\mathrm{x} + 2\mathrm{y}^2)\frac{\mathrm{dy}}{\mathrm{dx}}$
26. Verify whether $(\sin \mathrm{x} \sin \mathrm{y} - \mathrm{xe}^{\mathrm{y}})\mathrm{dy} = (\mathrm{e}^{\mathrm{y}} + \cos \mathrm{x} \cos \mathrm{y})\mathrm{dx}$ is exact and solve it.
27. Solve $(\mathrm{y}^2\mathrm{e}^{\mathrm{x}} + 2\mathrm{xy})\mathrm{dx} - \mathrm{x}^2\mathrm{dy} = 0$.
28. Solve $\mathrm{x}^2(\mathrm{y} - \mathrm{px}) = \mathrm{yp}^2$
29. Solve $(\mathrm{x}^2\mathrm{y} - 2\mathrm{xy}^2)\mathrm{dx} - (\mathrm{x}^3 - 3\mathrm{x}^2\mathrm{y})\mathrm{dy} = 0$
30. Solve $\mathrm{x} = \mathrm{y} + a\log p$

TEN MARK QUESTIONS

31. Solve $y[\mathrm{xy} + 2\mathrm{x}^2\mathrm{y}^2]\mathrm{dx} + \mathrm{x}[\mathrm{xy} - \mathrm{x}^2\mathrm{y}^2]\mathrm{dy} = 0$.
32. Solve $\mathrm{p}^2 + 2\mathrm{y}\cot \mathrm{xp} = \mathrm{y}^2$
33. Solve $\mathrm{p}^2 + \mathrm{pxy} = y^2 \log y$.

34. Solve $y = xp + x\sqrt{1 + \mathrm{p}^2}$.

35. Solve $p^2 + \left(x + y - 2\frac{y}{x}\right)p + xy + \frac{y^2}{x^2} - y - \frac{y^2}{x} = 0$.

36. Solve $x = 1 - \frac{p}{\sqrt{p^2+1}}$

37. Solve$y = 3\mathrm{x} + \log p$.

38. To check whether $e^y dx + xe^y dy + 2ydy = 0$ is exact and solve it.

39. Solve $y = -px + x^4p^2$

40. Solve $\mathrm{p} = \log(px - y)$

UNIT – II

CHOOSE THE CORRECT ANSWERS

1. How many types of second order differential equations?

A) 4

B) 7

C) 8

D) 6

2. The solution of auxillary equation $(\mathrm{D}^2 + 2\mathrm{D} + 2)\mathrm{Y} = 0$ is

A) $m^2 + 2m + 2 = 0$

B) $m^2 + 2 = 0$

C) $D^2 - 2m + 2 = 0$

D) $m^2 + 2m + 2$.

3. The solution of the roots $(\mathrm{D}^2 + 4\mathrm{D} + 4)\mathrm{Y} = 0$ is

A) $m_1 \neq m_2$

B) $m_1 = m_2$

C) $m = \alpha + i\beta$

` D) Not defined

4. The two roots are equal then the complementary function is

A) $(\mathrm{Ax} + \mathrm{B})\mathrm{e}^{\mathrm{mx}}$

B) $(\mathrm{Ax}+\mathrm{B})\mathrm{e}^{\mathrm{m}_1\mathrm{x}}$

C) $A\mathrm{e}^{\mathrm{m}_1\mathrm{x}}+\mathrm{Be}^{\mathrm{m}_2\mathrm{x}}$

D) $(\mathrm{Ax}+\mathrm{B})$

5. The value of $\cos A \sin B$ is

A) $\frac{1}{2}\{\cos(A+B)+\cos(A-B)\}$

B) $\frac{1}{2}\{\cos(A-B)-\cos(A+B)\}$

C) $\frac{1}{2}\{\cos(AB)+\sin(AB)\}$

D) $\frac{1}{2}\{\sin(A+B)-\sin(A-B)\}$

6. The solution of the particular integral $(\mathrm{D}^2-7\mathrm{D}+12)\mathrm{Y}=\mathrm{e}^{11\mathrm{X}}$ is

A) $\frac{e^{11x}}{88}$

B) $\frac{e^{72x}}{56}$

C) $\frac{-e^{-11x}}{5}$

D) $\frac{e^{11x}}{56}$

7. The value of $\cos^3\mathrm{x}$ is

A) $\frac{1-\cos 2x}{2}$

B) $\frac{1+\cos 2x}{2}$

C) $\frac{3\sin x-\sin 3x}{4}$

D) $\frac{3\cos x+\cos 3x}{4}$

8. The value of $(1+\mathrm{X})^{-2}$ is

A) $1-2X+3X^2-\cdots$

B) $1-X+X^2-\cdots$

C) $1+2X+3X^2+\cdots$

D) $1+X+X^2+\cdots$

9. The complementary function $(D^2 - 4D + 3)Y = 0$ is

A) 1, 3

B) 2, 4

C) 5, 3

D) 3, 1

10. The complementary function $\frac{d^2y}{dx^2} + 4y = 4\tan 2x$ is

A) $A\cos 2x + B\sin 2x$

B) $e^{\alpha x}[A\cos \beta x + B\sin \beta x]$

C) $c_1\cos 2x + c_2\sin 2x$

D) $c_1\sin 2x + c_2\cos 2x$

ANSWERS:

1. D 2.A 3.B 4. A 5.D 6.D 7.D 8.A 9.A 10.C

TWO MARK QUESTIONS

11. Find the C.F of $(D^2 + 1)Y = e^{2X}$.

12. Find the P.I of $(D^2 + 9)Y = \cos 4x$

13. Find the particular integral of $(D^2 + 16)Y = e^{-3X}$.

14. Find $(D^2 + 9D + 3)Y = e^{4x}$

15. Find the C.F of $(X^2D^2 - 3XD + 4)Y = X^2$

16. Write down the condition for complementary function?

17. Find $(D^2 + 7D + 14)y = 0$.

18. Find $(D^2 + 4)y = \sin 2x$.

19. To find complementary function of $(D^2 + 2D + 4)y = 0$.

20. Write down the formula for method of variation of parameters?

FIVE MARK QUESTIONS

21. Solve $(D^2 + 3D + 2)y = x^2$.

22. Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y + 37\sin 3x = 0$

23. Solve $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=e^x$.

24. Solve $(D^2+16)y=e^{-3x}+\cos 4x$

25. Solve $(D^2-4D+13)y=7x^2+11x+9$.

26. Solve $(D^2+3D+6)y=\cos 5x\cos 2x$

27. Solve $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=\sin\log x$

28. Solve $\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}=\frac{12\log x}{x^2}$

29. Solve $(D^2+1)y=\sec x$

30. Solve $(D^2+6D+8)y=\cos^2 x$.

TEN MARK QUESTIONS

31. Solve $(D^2-2D+1)y=xe^x\sin x$

32. Solve $(D^2-2D+3)y=e^x\cos 2x+2e^{2x}$

33. Solve $(D^2+1)y=x^2e^{2x}+x\cos x$

34. Solve $\frac{d^2y}{dx^2}+4y=4\tan 2x$ using method of variation of parameters.

35. Solve $(D^2-4D+3)y=\sin 3x\cos 2x$

36. Solve $(D^2-1)y=x^2\sin 2x$

37. Solve $(D^2+2D+4)y=e^{4x}+\cos^3 x+\sin 2x$

38. Solve $(D^2-2D+1)y=x\sin x$

39. Solve $(x^2D^2-3xD+4)y=x^2$

40. Solve $(D^2-1)y=x^2\cos 2x$

UNIT –III

CHOOSE THE COORRECT ANSWERS

1. The value of z=ax+by+a

A) px+qy+p

B) pxy-qx-p

C) px+qy-p

D) px+qy

2. If z=$e^y f(x+y)$then the value of P is

A) q-p

B) p+z

C) $e^y f'(x+y)$

D) p+q+z=0

3. The value of complete solution of $p^2+q^2=4$ is

A) $ax-\left(\sqrt{4-a^2}\right)y+c$

B) $ay-\left(\sqrt{4+a^2}\right)x+c$

C) $ax+\left(\sqrt{4-a^2}\right)y+c$

D) 0

4. The function of $f_1(x,p)=f_2(y,q)=a$ then the complete solution is

A) pdx+qdy

B) pdxqdy

C) $\int pdx+\int qdy$

D) dz=pdx+qdy

5. How many types of Lagrange's equations?

A) 1

B) 2

C) 3

D) None of these

6. If $(y^2+z^2)p-xyq=-xz$ then the value of c_1 is

A) yz

B) $\frac{y}{z}$

C) $\frac{x+y}{z}$

D) xy

7. The value of P, Q, and R is $(\mathrm{mz}-\mathrm{ny})\mathrm{p}+(\mathrm{nx}-\mathrm{lz})\mathrm{q}=\mathrm{lx}$

A) $(\mathrm{mz}-\mathrm{ny})(\mathrm{nx}+\mathrm{lz}), \mathrm{lx}$

B) $lx, (\mathrm{nx}+\mathrm{lz}), (\mathrm{mz}-\mathrm{ny})$

C) $lx, (\mathrm{mz}-\mathrm{ny})$

D)$(\mathrm{mz}-\mathrm{ny}), (\mathrm{nx}-\mathrm{lz}), \mathrm{lx}$

8. The value of R is $p\sin x+qy=4\tan x$

A) $\sin x$

B) qy

C) $4\tan x$

D)$\sin xy$

9. The value of p is px+qy-pq=0

A) pc

B) qc

C) pq=c

D) p=c

10. The value of $\mathrm{F_p}$ is $px+qy+\mathrm{p}^2+\mathrm{q}^2=z$

A) p

B) q

C) x+2p

D) x-2p

ANSWERS:

1. A 2.C 3.C 4.D 5.B 6.B 7.D 8.C 9.B 10.C

TWO MARK QUESTIONS

11. Solve $Z=(\mathrm{x}+\mathrm{a})(\mathrm{y}+\mathrm{b})$

12. Solve pq=k

13. Eliminate the function f from $z=\mathrm{e}^{\mathrm{y}}\mathrm{f}(\mathrm{x}+\mathrm{y})$.

14. Solve $\frac{\partial^2 z}{\partial x \partial y} = x^2 + y^2$

15. Eliminate the function f from $z = f(x^2+y^2)$

16. Define Lagrange's equations.

17. Define method of group.

18. Write down the formula for charpit's method?

19. Solve$z = ax + by + a^2+b^2$.

20. Solve $z = (x + y)f(x^2 - y^2)$

FIVE MARK QUESTIONS

21. Solve $(y^2+z^2)p - xyq = -xz$

22. Solve pq+p+q=0

23. Solve $\tan x \frac{\partial z}{\partial x} + \tan y \frac{\partial z}{\partial y} = \tan z$

24. Solve pxy+pq+qy=yz

25. Solve $p\cot x + q\cot y = \cot z$

26. Solve $p + q = \sin x + \sin y$

27. Solve $p^2+q^2 = npq$

28. Solve $\sqrt{p} + \sqrt{q} = 1$

29. Solve pq=1

30. Solve $px + qy + p^2+q^2 = z$

TEN MARK QUESTIONS

31. Solve $z = px + qy + 2\sqrt{pq}$

32. Solve $xp^2 - ypq + y^3q - y^2z = 0$

33. Solve $z = px + qy + \left(\frac{p}{q} - p\right)$

34. Solve $(mz - ny)p + (nx - lz)q = ly - mx$

35. Solve px+qy-pq=0

36. Solve $(q - p) + (x - y) = 0$

37. Solve $q = px + \mathrm{p}^2$

38. Solve $z = \mathrm{p}^2 + \mathrm{q}^2$

39. Solve $z = px + qy + \sqrt{1 + \mathrm{p}^2 + \mathrm{q}^2}$

40. Solve $(\mathrm{xz} + \mathrm{yz})\mathrm{p} + (\mathrm{xz} - \mathrm{yz})\mathrm{q} = \mathrm{x}^2 + \mathrm{y}^2$

UNIT - IV

CHOOSE THE CORRECT ANSWERS

1. The two roots are real and equal then the complementary function is

A) $z = \emptyset_1(\mathrm{y} + \mathrm{mx}) + \mathrm{x}\emptyset_2(\mathrm{y} + \mathrm{mx})$

B) $(\mathrm{Ax} + \mathrm{B})\mathrm{mx}$

C) $\emptyset_1(\mathrm{y} + \mathrm{m}_1\mathrm{x})$

D) $\emptyset_2(\mathrm{y} - \mathrm{ix})$

2. The auxillary equation is $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 5\frac{\partial^2 z}{\partial y^2} = 0$.

A) $\mathrm{m}^2 - 4\mathrm{m}^2 + 5m = 0$

B) $\mathrm{m}^2 - 4\mathrm{m}^4 + 5m = 0$

C) $\mathrm{m}^2 - 4m + 5 = 0$

D) $\mathrm{m}^2 - m + 5 = 0$

3. The two roots are unequal then the complementary function is

A) $z = (\mathrm{Ax} + \mathrm{B})\mathrm{e}^{\mathrm{mx}}$

B) $A\mathrm{e}^{\mathrm{m}_1\mathrm{x}+\mathrm{B}}\mathrm{e}^{\mathrm{m}_2\mathrm{x}}$

C) z=0

D) $z = \emptyset_1(\mathrm{y} + \mathrm{m}_1\mathrm{x}) + \emptyset_2(\mathrm{y} + \mathrm{m}_2\mathrm{x})$

4. The solution of particular integral is $\frac{\partial^2 z}{\partial x^2} - 3\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 2e^{3x+4y}$

A) e^{3x+4y}

B) $\frac{2}{5}e^{3x+4y}$

C) 0

D) $\frac{2}{5}e^{3x-4y}$

5. If $D^2 - 9D'^2 = xy$ then the value of complementary function is

A) $\emptyset_1(y-3x) + \emptyset_2(y-3x)$

B) $\emptyset_1(y+3x) + \emptyset_2(y+3x)$

C) $\emptyset_1(y-13x) + \emptyset_2(y+x)$

D) $\emptyset_1(y+3x) + \emptyset_2(y-3x)$

6. If $(D+D')^2 = e^{x-y}$ then the roots are

A) $m_1 \neq m_2$

B) $m = \pm i$

C) $m_1 = m_2$

D) $m_1 = -m_2$

7. If $[D^2 - 2DD' + D'^2]z = \cos(x-3y)$Then the particular integral value is

A) $-\frac{1}{16}\cos(x-3y)$

B) $\frac{1}{169}\cos(x-3y)$

C) 0

D) $\frac{1}{58}\cos x$

8. If $(D^2 + DD' - 6D'^2)z = \cos(2x+y)$ then the C.F is

A) $m_1 \neq m_2$

B) $m_1 = m_2$

C) $m = \pm ix$

D) $\emptyset_1(y+2x) + \emptyset_2(y-3x)$

9. If $(D^3 + D^2D' - DD'^2 - D'^3)z = 0$ Then the value of auxillary equation is

A) $m^2 + 2m + 1 = 0$

B) $m^3 + m^2 - m - 1 = 0$ \

C) $m^2 - m - 1 = 0$

D) 0

10. If $(\mathrm{D}^2 + \mathrm{DD}')\mathrm{z} = \mathrm{e}^{\mathrm{x}-\mathrm{y}}$ then the Solution of particular integral is

A) $x\mathrm{e}^{\mathrm{x}-\mathrm{y}}$

B) $xy\mathrm{e}^{\mathrm{x}-\mathrm{y}}$

C) $\frac{\mathrm{e}^{\mathrm{x}-\mathrm{y}}}{12}$

D) $\mathrm{e}^{\mathrm{x}-\mathrm{y}} + 56$

ANSWERS:

1. A 2.C 3.D 4.B 5.C 6.C 7.A 8.D 9.B 10.A

TWO MARK QUESTIONS

11. Define homogeneous and non-homogeneous linear equations.

12. Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 2\frac{\partial^2 z}{\partial y^2} = 0.$

13. Write down the condition for complementary function?

14. Find the particular integral of $\left(\mathrm{D}^2 - \mathrm{D}'^2\right)\mathrm{z} = \mathrm{e}^{\mathrm{x}+\mathrm{y}}$

15. Find the complementary function of $\left(\mathrm{D}^2 + 3\mathrm{DD}' + 2\mathrm{D}'^2\right)\mathrm{z} = 0.$

16. Solve $\left(9\mathrm{D}^2 + 6\mathrm{DD}' + \mathrm{D}'^2\right)\mathrm{z} = 0.$

17. Solve $(\mathrm{D}^2 + \mathrm{DD}')z = e^{x-y}$

18. Find the particular integral of $\left(\mathrm{D}^2 - 9\mathrm{D}'^2\right)\mathrm{z} = \mathrm{xy}$

19. To find complementary function of $\left(\mathrm{D}^3 + \mathrm{D}^2\mathrm{D}' + 8\mathrm{D}'^2 + 8\mathrm{D}'^3\right)z = 0$

20. Define general homogeneous linear equation.

FIVE MARK QUESTIONS

21. Solve $\left(\mathrm{D}^2 - 3\mathrm{DD}' + 2\mathrm{D}'^2\right)\mathrm{z} = \mathrm{xy}.$

22. Solve $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = -4\pi(x^2 + y^2)$

23. Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 12(x + y)$

24. Solve $\left(\mathrm{D}^2 - 2\mathrm{DD}' + \mathrm{D}'^2\right)\mathrm{z} = \cos(\mathrm{x} - 3\mathrm{y})$

25. Solve $\left(\mathrm{D}^2 - 2\mathrm{DD}' + 2\mathrm{D}'^2\right)\mathrm{z} = \sin(\mathrm{x} - \mathrm{y})$

26. Solve $\left(\mathrm{D}^2 - 4\mathrm{D}'^2\right)\mathrm{z} = \cos 4\mathrm{x} \cos 3\mathrm{y}$

27. Solve $\frac{\partial^2 z}{\partial x^2} - 3\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 2e^{3x+4y}$

28. Solve $\frac{\partial^2 z}{\partial x^2} + 4\frac{\partial^2 z}{\partial x \partial y} - 5\frac{\partial^2 z}{\partial y^2} = x + y^2.$

29. Solve $\frac{\partial^2 z}{\partial x^2} - a^2\frac{\partial^2 z}{\partial y^2} = x^2$

30. Solve $\left(\mathrm{D}^2 + \mathrm{DD}' - 6\mathrm{D}'^2\right)\mathrm{z} = \cos(2\mathrm{x} + \mathrm{y})$

TEN MARK QUESTIONS

31. Solve $\left(\mathrm{D}^2 + \mathrm{DD}' - 6\mathrm{D}'^2\right)\mathrm{z} = \cos(2\mathrm{x} + \mathrm{y}) + \mathrm{e}^{\mathrm{x}-\mathrm{y}}$

32. Solve $\left(\mathrm{D}^2 + 4\mathrm{DD}' - 5\mathrm{D}'^2\right)\mathrm{z} = 3e^{2x-y} + \sin(x - 2y)$

33. Solve $\left(\mathrm{D}^2 - 2\mathrm{DD}' + \mathrm{D}'^2\right)\mathrm{z} = \mathrm{x}^2\mathrm{y}^2\mathrm{e}^{\mathrm{x}+\mathrm{y}}$

34. Solve $\left(\mathrm{D}^3 + \mathrm{D}^2\mathrm{D}' - \mathrm{DD}'^2 - \mathrm{D}'^3\right)z = e^x \cos 2y$

35. Solve $\left(\mathrm{D}^2 - \mathrm{D}'^2\right)\mathrm{z} = \mathrm{e}^{\mathrm{x}-\mathrm{y}} \sin(2\mathrm{x} + 3\mathrm{y})$

36. Solve $\left(\mathrm{D}^2 - 3\mathrm{DD}' + 2\mathrm{D}'^2\right)\mathrm{z} = \sin x \cos y$

37. Solve $\left(\mathrm{D}^2 - \mathrm{DD}' - 6\mathrm{D}'^2\right)\mathrm{z} = \mathrm{x} + \mathrm{y}$

38. Solve $\left(\mathrm{D}^2 - 7\mathrm{DD}' - 6\mathrm{D}'^3\right)\mathrm{z} = \mathrm{x}^2 + \mathrm{xy}^2 + \mathrm{y}^3 + \cos(x - y)$

39. Solve $\left(\mathrm{D}^2 + \mathrm{D}'^2\right)\mathrm{z} = \mathrm{x}^2\mathrm{y}^2$

40. Solve $(\mathrm{D}^2 + \mathrm{DD}')\mathrm{z} = \cos 2\mathrm{x} \cos \mathrm{y}$

UNIT- V

CHOOSE THE CORRECT ANSWERS

1. The solution of $L\{\mathrm{e}^{-\mathrm{at}}\}$ is

A) $\frac{1}{s+a}$

B) $s - a$

C) $\frac{1}{s-a}$

D) $s - 6a$

2. The value of $L\{\cos \text{hat}\}$ is

A) $\frac{a}{s^2-a^2}$

B) $\frac{s}{s^2-a^2}$

C) $\frac{1}{s^2+a^2}$

D) $\frac{1}{s^2-a^2}$

3. The value of $L\{\mathrm{t}^{\mathrm{n}}\}$ is

A) $n!$

B) $\Gamma n+1$

C) $\frac{n!}{s^n}$

D) $\frac{n!}{s^{n+1}}$

4. The value of $L\{\mathrm{f(t)}\}$ is

A) $\int_0^\infty e^{-st} f(-t)dt$

B) $\int_0^1 e^{-st} f(t)dt$

C) $\int_0^\infty e^{-st} f(t)dt$

D) 0

5. If $L\{\mathrm{f(t)}\} = \mathrm{F(s)}$ and if $\frac{f(t)}{t}$ has a limit at $t \to 0$ then $L\left\{\frac{\mathrm{f(t)}}{\mathrm{t}}\right\}$ is

A) $\int_0^\infty F(s)ds$

B) $\int_0^1 F(s)ds$

C) $\int_0^\infty F(t)dt$

D) $\int_s^\infty F(s)ds$

6. The value of $L\{\mathrm{e}^{-\mathrm{at}} \sin \mathrm{bt}\}$ is

A) $\frac{\mathrm{b}}{(\mathrm{s+a})^2+\mathrm{b}^2}$

B) $(s+a)^2+b^2$

C) $\frac{s+a}{s-a}$

D) $\frac{n!}{s^{n+1}}$

7. The solution of $L^{-1}\left\{\frac{n!}{s^{n+1}}\right\}$ is

A) 0

B) ∞

C) t^n

D) t^∞

8. The value of $L^{-1}\left\{\frac{1}{(s-3)^5}\right\}$ is

A) $\frac{e^{3t}t^4}{24}$

B) $\frac{e^{3t}t^5}{24}$

C) $\frac{e^{2t}t^4}{24}$

D) $\frac{e^{t}t^4}{20}$

9. The value of $L^{-1}\left\{\frac{a}{s^2a^2+b^2}\right\}$ is

A) $\frac{1}{b}\sin(bt)$

B) $\frac{1}{b}\sin\left(\frac{bt}{a}\right)$

C) $\sin\left(\frac{bt}{a}\right)$

D) $\frac{1}{b}$

10. If $L\{f(t)\}=F(s)$ then $L\{tf(t)\}$ is

A) $F'(s)$

B) $F'(t)$

C) $-F'(s)$

D) $-tf(t)$

ANSWERS:

1. A) 2.B) 3.D) 4.C) 5.D) 6.A) 7.C) 8.A) 9.B) 10.C)

TWO MARK QUESTIONS

11. Find $L\{\sin at\}$

12. Find $L\{e^{-at}\}$

13. Define Laplace transforms.

14. Show that $L\{c.f(t)\} = c.L\{f(t)\}$.

15. Find $L\{te^{-at}\}$

16. Find $L\{e^{-at}.t^n\}$

17. Find $L\{e^{at}\cos bt\}$

18. Find $L^{-1}\left\{\frac{1}{(s+a)^2}\right\}$

19. Find $L\{\cos hat\} = \frac{s}{s^2-a^2}$

20. Find $L^{-1}\left\{\frac{a}{s^2a^2+b^2}\right\}$

FIVE MARK QUESTIONS

21. Evaluate $L^{-1}\left\{\frac{5s+3}{(s-1)(s^2+2s+5)}\right\}$

22. Solve $L\{\sin^2 2t\}$

23. Evaluate $\int_0^\infty e^{-2t}\sin 3t\,dt$

24. Solve $L^{-1}\left\{\frac{a}{s^2a^2-b^2}\right\}$

25. Prove that $L\{f'(t)\} = s.L\{f(t)\} - f(0)$.

26. Solve $L\{\cos^3 2t\}$

27. Evaluate $L^{-1}\left\{\frac{1}{s^2(s+1)}\right\}$

28. Solve $\mathrm{L}^{-1}\left\{\frac{1}{\mathrm{s}(\mathrm{s}^2+\mathrm{a}^2)}\right\}$

29. Solve $\int_0^\infty e^{-t}\left(\frac{sin^2 t}{t}\right)dt$

30. Evaluate $\int_0^\infty \left(\frac{e^{-t}-e^{-2t}}{t}\right)dt$

TEN MARK QUESTIONS

31. Solve $\frac{\mathrm{d}^2\mathrm{y}}{\mathrm{dt}^2}+\frac{2\mathrm{dy}}{\mathrm{dt}}-3\mathrm{y}=\mathrm{sint}$ given that $y=\frac{\mathrm{dy}}{\mathrm{dt}}=0$ when $t=0$

32. Evaluate $L\{\mathrm{f(t)}\}$ when $\mathrm{f(t)}=\begin{cases}(\mathrm{t}-1)^2, & t>1\\ \mathrm{o}, & t<1\end{cases}$

33. If $L\{\mathrm{f(t)}\}=\mathrm{F(s)}$ then $L\{\mathrm{tf(t)}\}=-\frac{\mathrm{d}}{\mathrm{ds}}F(s)$

34. Evaluate $L\{\mathrm{tcos}^2\mathrm{t}\}$

35. Evaluate $\int_0^\infty \left(\frac{\cos 6t-\cos 4t}{t}\right)dt$

36. Evaluate $L^{-1}\left\{\frac{1}{\mathrm{s}(\mathrm{s}+2)^3}\right\}$

37. Evaluate $L^{-1}\left\{\frac{1}{(\mathrm{s}^2+9)^2}\right\}$

38. The method of partial function can be used to find inverse transform of certain function then evaluate $\mathrm{L}^{-1}\left\{\frac{1}{\mathrm{s}(\mathrm{s}+1)(\mathrm{s}+2)}\right\}$

39. Solve $\mathrm{L}^{-1}\left\{\frac{2\mathrm{s}-1}{\mathrm{s}^2(\mathrm{s}-1)^2}\right\}$

40. Solve $\mathrm{L}^{-1}\left\{\frac{1}{(\mathrm{s}^2+\mathrm{a}^2)^2}\right\}$