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QUESTION BANK

Title of the Paper

GRAPH THEORY

COURSE – III B.Sc., Maths

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MAJOR BASED ELECTIVE II (A)

GRAPH THEORY

Objectives:

1. To introduce the notion of graph theory and its applications.
2. To learn the techniques of combination in Graph Theory.

UNIT I

Introduction - The Konigsberg Bridge Problem - Graphs and subgraphs: Definition and Examples - Degrees - Subgraphs - Isomorphism. –independent sets and coverings.

UNIT II

Matrices - Operations on Graphs - Walks, Trails and Paths – Connectedness and Components - Eulerian Graphs.

UNIT III

Hamiltonian Graphs (Omit Chavatal Theorem) - Characterization of Trees - Centre of a Tree.

UNIT IV

Planarity: Introduction - Definition and Properties - Characterization of Planar Graphs.

UNIT V

Directed Graphs: Introduction - Definitions and Basic Properties – Some Applications: Connector Problem - Kruskal's algorithm - Shortest Path Problem – Dijkstra's algorithm.

Textbook

1. S. Arumugam and S. Ramachandran, Invitation to Graph Theory, SciTech Publications (India) Pvt. Ltd., Chennai, 2006.

References

1. Narsingh Deo, Graph Theory with applications to Engineering and Computer Science, Prentice Hall of India, 2004.

2. Gary Chartrand and Ping Zhang, Introduction to Graph Theory, Tata McGraw-Hill Edition, 2004.

GRAPH THEORY

UNIT – I

CHOOSE THE CORRECT ANSWER

1. Which is one of the using for graph theory?

A) Physics and genetics

B) Dynamics

C) Genetics

D) None of these

2. Who is first written by graph theory?

A) Euler

B) Choudam

C) H.B.Waliker

D) A.R.Rao

3. If $X = \{(a, b),\ (a, c), (a, d)\}$ then the graph represented by adjacent

A) Non- adjacent

B) adjacent

C) A& B

D) none

4. Let $X = \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$it's represented by

A) Graph

B) Self loop

C) parallel edges

D) complete graph

5.If $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{e_1, e_2, e_3, e_4, e_5\}$ then it's called a ------- graph

A) Pseudo graph

B) non complete

C) Multi graph

D) B & C

6. A regular graph of degree 3 is called

A) Cubic graph

B) isolated graph

C) regular graph

D) none of these

7. Which is the following statements are true?

A) Every cubic graph has an even number of points.

B) Every regular graph has an even number of points.

C) None of these

D) A & B true

8. Any self complementary graph has an 4n

A) Above statements are true

B) not true

C) Contradiction

D) none

9. The following graph of self loops and parallel edges?

A) Complete graph

B) simple graph

C) null graph

D) general graph

10. Every edges in a graphs are represented by

A) Curve

B) straight line

C) long

D) all the above

ANSWERS:

1. A 2. A 3. B 4. D 5. C 6. A 7. D 8. A 9. D 10. D

TWO MARK QUESTIONS

11. Define graph.

12. Define simple graph.

13. Define complete graph.

14. What is a degree.

15. Define end vertices.

16. Define a loop (or) self loop.

17. Define parallel edges.

18. Define Peterson graph.

19. Let G be a K- regular graph with bipartition$(\mathrm{v}_1, \mathrm{v}_2)$ and k>0 prove that$|\mathrm{v}_1| = |\mathrm{v}_2|$.

20. Define bigraph.

FIVE MARK QUESTIONS

21. Explain regular graph?

22. Explain isolated graph?

23. The number of vertices of odd degree in a graph G is always even.

24. Prove $\Gamma \mathrm{G} = \Gamma \overline{\mathrm{G}}$

25. A set $S \subseteq V$is an independent set of G iff V-S is a covering of G

26. To prove that $\alpha + \beta = P$

27. Explain finite and infinite graph?

28. Explain sub graphs?

29. Let $V = \{1, 2, 3, 4\}$ $\mathrm{E} = \{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}, \mathrm{e}, \mathrm{f}\}$ to find a graph?

30. The graph Let V=10; E=15 to find the graph?

TEN MARK QUESTIONS

31. The maximum number of lines among all P points with no triangles is $\frac{\mathrm{P}^2}{4}$

32. Explain isomorphism and given an examples

33. Show that in any group of 2 (or) more people there are always 2 with exactly the same no of friends inside the group.

34. Explain the Konigsberg bridge problem.

35. Explain edge disjoint and vertex disjoint.

36. The maximum degree of any vertices in a simple graph G with n vertices is n-1

37. The sum of the degrees of all vertices in a graph G is always even.

38. Explain incidents and degree

39. Explain the operations of graph

40. Explain automorphism, complement, and Ulam's conjecture.

UNIT - II

CHOOSE THE CORRECT ANSWERS

1. Which is one of the another name of a walk is

A) Terminals

B) trail

C) graph

D) tree

2. Which graph is beginning and ending of the same vertex?

A) Walk

B) open walk

C) closed walk

D) none of these

3. Which is the following statement are true?

A) A self loop can be included in a walk but not in a path.

B) A self loop can be included in a walk also referred to path.

C) A & B

D) None of these

4. A circuit is also called a____?

A) Triangle

B) curve

C) cycle

D) line

5. Which are the following statements are true

A) A closed walk of odd length contains a cycle.

B) An open walk of even length contains a cycle.

C) A closed walk of even length contains a cycle.

D) None

6. Let $V = \{v_1, v_2\} \& E = \{e_1\}$ this graph is

A) Tree

B) null graph

C) connected graph

D) walk

7. Which are the following statements are correct?

A) None of these

B) If G is disconnected then $\overline{G}$ is connected.

C) If $\overline{G}$ is connected then g is disconnected.

D) B & C

8. Which graph is known as star of Davit?

A) Walk

B) Euler graph

C) path

D) tree

9. In a graph G any u-v walk contains a ____ path.

A) u-v

B) v-u

C) u+v

D) v+u

10. When the graph is a path of length k.

A) $\delta \leq K$

B) $\delta < K$

C) $\delta \geq K$

D) None

ANSWERS:

1. B 2. C 3. A 4. C 5. A 6. C 7. D 8. B 9. A 10. C

TWO MARK QUESTIONS

11. Define adjacency matrix.

12. Define incidence matrix.

13. Define terminal vertices.

14. Define open walk.

15. Define connected graph.

16. Define cut point.

17. Define Eulerian graph.

18. Define cycle.

19. Define bridge.

20. Define components.

FIVE MARK QUESTIONS

21. Explain path with given an examples

22. Explain walk given an examples

23. In a graph G any u-v walk contains a u-v path

24. If $\delta \geq K$ then G has a path of length K.

25. Explain circuit?

26. A graph G with P points and $\delta \geq \frac{P-1}{2}$ is connected.

27. If G not connected then $\overline{G}$ is connected.

28. Every non-trivial connected graph has at least 2 points which are not cut points.

29. Explain connected graph?

30. A closed walk of odd length contains a cycle?

TEN MARK QUESTIONS

31. Let G_1 be a (p_1, q_1) graph and G_2 be a (p_2, q_2) graph then

I) $(G_1 \cup G_2)$ is a $(p_1 + p_2, q_1 + q_2)$ graph

II) $(G_1 + G_2)$ Is a $(p_1 + p_2, q_1 + q_2 + p_1 p_2)$ graph

III) $(G_1 \times G_2)$ Is a $(p_1 p_2, q_1 p_2 + q_2 p_1)$ graph

32. If A is the adjacent matrix of a graph with $V = \{v_1, v_2, \ldots . v_p\}$ prove that for any $n \geq 1$ the $(i, j)^{th}$ entry of A^n is the no of $v_i - v_j$ walk o length n in G

33. In a connected (or) disconnected graph G has exactly two vertices of odd degree then there exists a path between these two vertices.

34. A graph G is connected iff for any partition of V into subsets v_1 and v_2 there is a line of G joining a point of v_1 to a point of v_2

35. A graph G with at least two points is bipartite iff all its cycle are of even length

36. A line X of a connected graph G is a bridge iff X is not on any cycle of G.

37. A given connected graph G is an Euler graph iff all the vertices of G are of even degree.

38. A connected graph G is an Euler graph iff it can be decomposed into circuits.

39. Let V be a point of a connected graph G the following statements are equivalent

I) v is a cut point of G

II) There exists partition of v-{v} into subsets U & W. Such that for each $u \in U$ and $w \in W$ the point v is on every u-w path.

III) There exists two points' u & w distinct from v such that v is a u-w path.

40. Explain the types of walk?

UNIT – III

CHOOSE THE CORRECT ANSWER

1. Who is introduced by Hamiltonian Graph?

A) Sir. William Hamilton

B) S.S Sastry

C) Euler

D) A & C

2. Which year introduced by Hamiltonian graph?

A) 1851

B) 1856

C) 1859

D) 1818

3. Which year introduced by tree?

A) 1920

B) 1858

C) 1817

D) 1857

4. Who is introduced by tree?

A) R.Balakrishnan

B) H.B.Waliker

C) Cayley

D) B & C

5. How many vertices and edges in Hamiltonian circuits?

A) n by n

B) m by n

C) n by n-1

D) none

6. If we remove any one edge from Hamiltonian circuits the it's called ----------?

A) Circuits

B) path

C) centre

D) none

7. Let $V = \{v_1\}$ if the graph is?

A) Finite graph

B) connected graph

C) tree

D) simple graph

8. Which are the following statements are true?

A) A tree with n vertices and n-1 edges.

B) A tree has n-1 vertices and n edges.

C) Every tree has either one centre.

D) None of these.

9. What is another name of centre?

A) Bicentre

B) eccentricity

C) adjacent

D) radius

10. Which are following statements is correct?

A) Every disconnected graph has at least one spanning tree.

B) Every connected graph has at least one spanning tree.

C) The number of vertices in a binary tree is always even.

D) Every binary tree is a spanning tree.

ANSWERS:

1. A 2. C 3. D 4. C 5. A 6. B 7. C 8. A 9. A 10.

TWO MARK QUSTIONS

11. Define Hamiltonian circuits.

12. Define tree.

13. Define sorting tree.

14. Define distance.

15. Define centre.

16. Define Eccentricity.

17. Define rooted tree.

18. Define binary tree.

19. Define spanning tree.

20. Define radius.

FIVE MARK QUESTIONS

21. There is one and only one path between every pair of vertices in a tree T.

22. In a graph G there is one and only one path between every pair of vertices G is a tree

23. Every binary tree is a rooted tree

24. Explain the property of binary tree

25. Every Hamiltonian graph is 2-connected

26. If G is Hamiltonian then for every non empty proper subset S of $V(\mathrm{G}), \omega(\mathrm{G}-\mathrm{S}) \leq |\mathrm{S}|$ where $\omega(\mathrm{H})$denotes the number of components in any graph H

27. Every tree has a centre consisting of either one point or two points are adjacent

28. Every connected graph has a spanning tree

29. A graph is Hamiltonian iff its closure is Hamiltonian

30. Explain path given an examples

TEN MARK QUESTIONS

31. In a complete graph with n vertices there are $\frac{n-1}{2}$ edges disjoint Hamiltonian circuits if n is odd number $n \geq 3$

32. State and prove Dirac's theorem

33. A tree with n vertices and has n-1 edges

34. The number of labeled tree with n vertices $(n \geq 2)$ is n^{n-2}

35. Show that the Peterson graph is non Hamiltonian

36. Let G be a (p, q) graph then the following statements are equivalent

1. G is a tree.
2. Every two points of G are joined by a unique path.
3. G is connected and p=q+1.
4. G is cyclic and p=q+1.

37. Explain the characterisation of trees

38. Any connected graph with n vertices n-1 edges is a tree .

39. A graph G with n vertices n-1 edges and no circuits is connected

40. The distance between vertices of a connected graph of a metric

UNIT - IV

CHOOSE THE CORRECT ANSWER

1. The $\pi - G$ is the union of disjoint regions such regions are called ____

A) Faces

B) plane graph

C) planar

D) none

2. The k_5 is

A) Planar

B) non- planar

C) faces

D) none

3. Every planar graph embedded in a plane such that all edges are ___ segments.

A) Circle

B) polyhedral

C) straight line

D) curve

4. A graph is polyhedral iff its planar and ___?

A) 2-connected

B) 3- connected

C) 1-connected

D) 0-connected

5. If G is a connected plane graph having V,E and F as the sets of vertices , edges, faces respectively, then ___?

A) $|V| - |E| + |F| = 2$

B) $|V| - |E| + |F| = 8$

C) $|V| - |E| + |F| = 4$

D) $|V| - |E| + |F| = 9$

6. If G is a plane (p,q) graph with r faces and k components then____?

A) p-q+r=k+1

B) p-q-r=k=2

C) p+q+r=k+1

D) none of these

7. The $K_{3,3}$ is

A) Planar

B) none of these

C) non planar

D) face

8. Every planar graph has

A) genus 4

B) genus 2

C) genus 6

D) genus 0

9. If G is a maximal planar (p, q) graph without triangles and $p \geq 3$ then ---------?

A) None of these

B) $q \leq 2p - 4$

C) $q > 2p - 4$

D) $q = 2p - 4$

10. Every planar graph G with $p \geq 3$ vertices has at least three points of degree less than___?

A) 8

B) 0

C) 6

D) A & C

ANSWERS:

1. A 2. B 3. C 4. B 5. A 6.A 7. C 8. D 9. B 10.C

TWO MARK QUESTIONS

11. Define embedded.

12. Define planar.

13. Define faces.

14. Define exterior face.

15. Define maximal planar.

16. Define interior face.

17. Define triangulated graph.

18. Define homeomorphism.

19. Define subdivided.

20. Define plane graph.

FIVE MARK QUESTIONS

21. K_5 Is non planar?

22. If G is a plane (p,q) graph with r faces k components then p-q+r=k+1

23. If G is a (p, q) plane graph in which every face is an n cycle then $q = \frac{n(p-2)}{n-2}$

24. If G is a plane connected (p, q) graph without triangles and $p \geq 3$ then $q \leq 2p - 4$

25. The graph K_5 and K_3 is not planar

26. Every planar graph G with $p \geq 3$ vertices has at least three points of degree less than 6.

27. If $a(\mathrm{p}_1, \mathrm{q}_1)$ graph and $a(\mathrm{p}_2, \mathrm{q}_2)$ graph are homeomorphism then $\mathrm{p}_1 + \mathrm{q}_2 = \mathrm{p}_2 + \mathrm{q}_1$

28. Explain outer planar and maximal planar

29. Explain thickness and crossing number and genus

30. Explain elementary contradiction and geometric dual

TEN MARK QUESTIONS

31. Prove that if G is a connected plane graph having V, E and F as the sets of vertices, edges and faces respectively then $|\mathrm{V}| - |\mathrm{E}| + |\mathrm{F}| = 2$

32. Prove that a graph can be embedded in the surface of a sphere iff it can be embedded in a plane.

33. Every polyhedron has at least two faces with the same number of edges on the boundary.

34. Every 2-connected plane graph can be embedded in the plane so that any specified face is the exterior face.

35. In any connected plane (p, q) graph $(\mathrm{p} \geq 3)$ with r faces $q \geq \frac{3\mathrm{r}}{2}$ and $q \leq 3p - 6$

36. Every planar graph G with at least 3 points is a sub graph of a triangulated graph with the same number of vertices.

37. A graph is planar iff it has no sub graph homeomorphic to $\mathrm{K}_5(\mathrm{or})\mathrm{K}_{3,3}$.

38. A graph is planar iff it doesn't have a sub graph contradictable to $\mathrm{K}_5(\mathrm{or})\mathrm{K}_{3,3}$.

39. Show that there are no map five regions in the plane such that every pair of regions is adjacent.

40. Explain planar, embedded, faces.

UNIT – V

CHOOSE THE CORRECT ANSWERS

1. A weight digraph D is Eulerian iff every point of D has equal

A) In degree & out degree

B) out degree

C) in degree

D) all the above

2. A digraphs is called Eulerian if it has an____

A) trail

B) faces

C) Eulerian trail

D) A & C

3. The in degree $d^{-}(v)$ of a vertex v in a graph D is the number of arc having v as it's?

A) Terminal

B) initial

C) isomorphism

D) digraph

4. If two graphs are isomorphic then corresponding points have the___ pair

A) Out degree

B) in degree

C) same degree

D) none

5. The out degree $d^{+}(v)$ of v is the number of arcs having v as it's?

A) C & D

B) arcs

C) tail

D) initial

6. A digraphs are called functional if every point has out degree

A) 4

B) 1

C) 0

D) 6

7. A complete digraph has n vertices then it has___?

A) n-1arcs

B) n(n-1) arcs

C) n arcs

D) none of these

8. A walk in which the origin and terminus coincide is called

A) Open walk

B) path

C) closed walk

D) none

9. A walk in which all the vertices are distinct then it's called

A) Directed path

B) path

C) A & B

D) component

10. A nontrail closed walks whose origin and internal vertices are distinct then it's called

A) None of these

B) directed cycle

C) B & D

D) circuit

ANSWERS:

1. D 2. C 3. A 4. C 5. A 6. B 7. B 8. C 9. C 10. C

TWO MARK QUESTIONS

11. Define isomorphism.

12. Define weight of a graph.

13. Define sub diagraph.

14. Define underlying graph.

15. Define in degree.

16. Define out degree.

17. Define directed graph.

18. Define functional.

19. Define complete.

20. Define converse digraph.

FIVE MARK QUESTIONS

21. If two digraphs are isomorphic then corresponding points have the same degree pair.

22. In a digraphs D sum of the in degrees of all the vertices is equal to the sum of their out degrees each sum being equal to the number of arcs in D.

23. Explain connector problem?

24. Explain the algorithm of Kruskal's?

25. Explain the algorithm of Dijikstra's?

26. Explain the directed graph, in degree and out degree?

27. Explain the digraph?

28. Explain the basic properties of directed graphs.

29. Explain the isomorphism.

30. Explain converse digraph, complete, functional.

TEN MARK QUESTIONS

31. Explain the digraphs and it's not isomorphic with given an examples.

32. Explain the Kruskal's algorithm with given an example.

33. Explain shortest path problem.

34. Explain shortest distance and given an any examples?

35. If two digraphs are isomorphic then corresponding points have the same degree pair.

36. Explain digraph and its properties.

37. Explain the tail, head and its degree pair.

38. Explain isomorphism.

39. Explain the converse digraph, functional, complete

40. Find a minimum weight spanning tree when the distances between the points are 6.