ஸ்ரீ-ல-ஸ்ரீ காசிவாசி சுவாமிநாத சுவாமிகள் கலைக் கல்லூரி

திருப்பனந்தாள் – 612504

S.K.S.S ARTS COLLEGE, THIRUPPANANDAL - 612504

QUESTION BANK

Title of the Paper

INTEGRAL CALCULUS

COURSE – I B.Sc., Maths

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CORE COURSE II

INTEGRAL CALCULUS

Objectives:

1. To inculcatethe basics of integration and their applications.
2. To study some applications of definite integrals.
3. To understand the concepts of Beta, Gamma functions

UNIT I :

Revision of all integral models –simple problems

UNIT II :

Definite integrals -Integration by parts & reduction formula

UNIT III :

Geometric Application of Integration-Area under plane curves: Cartesian co-ordinates -Area of a closed curve -Examples -Areas in polar co-ordinates.

UNIT IV :

Double integrals –changing the order of Integration –Triple Integrals.

UNIT V :

Beta & Gamma functions and the relation between them–Integrationusing Beta & Gamma functions

UNIT – I

CHOOSE THE CORRECT ANSWER:

1.Find the value of $\int 2x\ dx$

a) x^2

b) x^3

c) x^4

d) $2x^2$

2. Find the value of $\int x^{\frac{3}{2}}\ dx$

a) $\frac{2x^{3/2}}{3}$

b) $\frac{2x^{4/2}}{4}$

c) $\frac{2x^{5/2}}{5}$

d) None of these

3. Find the value of $\int(2+3e^x)dx$

a) $2x^2+3e^x$

b) $2x+3e^x$

c) $2x^3+3e^x$

d) $2x^4+3e^x$

4. If $\int x^n dx$ is

a) $\frac{x^n}{n}$

b) $\frac{x^{n+1}}{n+1}$

c) $\frac{x^{n+2}}{n+2}$

d) None of these

5. If $\int tanx\ dx$ is

a) $\sin x$

b) $\cos x$

c) $\sin x\ \ cosx$

d) $\log \sec x$

6. Find the value of $\int \frac{1}{1+sinx}\ dx$ is

a) $\sin x - \cos x$

b) $\tan x - secx$

c) $\text{sce}\ \ x + \tan x$

d) None of these

7. Find the value of $\int x^{-4}\ dx$ is

a) $-x$

b) $\frac{-x^{-2}}{2}$

c) $\frac{-x^{-3}}{3}$

d) None of these

8. If $\int \frac{1}{\sqrt{1-x^2}} dx$

a) $\sin^{-1}x$

b) $\cos^{-1}x$

c) $\tan^{-1}x$

d) None of these

9. If $\int \frac{f'(x)}{f(x)}\ dx$ is

a) $\log(f'(x))$

b) $\log f(x)$

c) $-\log f(x)$

d) None of these

10. Find the value of $\int \frac{1}{1-cosx} dx$ is

a) $-\cot x - cosec\, x$

b) $\cot x - cosec\, x$

c) $\cot x + cosec\, x$

d) None of these

ANSWERS:

1) a 2) c 3) b 4) b 5) d 6) b 7) c 8) a 9) b 10) a

TWO MARK QUESTIONS

11. Evaluate $\int \left(ax + \frac{b}{x^2}\right) dx$.

12. Evaluate $\int \left(x^{\frac{2}{5}} - x^{\frac{-3}{5}}\right)^2 dx$.

13. Evaluate $\int \left(x + \frac{1}{x}\right)^2 dx$.

14. Evaluate $\int x^2(1-x)^2 dx$.

15. Evaluate $\int \left(x^2 - x^{\frac{-3}{5}}\right)^2 dx$.

16. Evaluate $\int \frac{1}{(x+7)^4} dx$.

17. Evaluate $\int (3-2x)^3 dx$.

18. Evaluate $\int \sqrt{x}\, dx$.

19. Evaluate $\int \frac{1}{(4-3x)^4} dx$.

20. Evaluate $\int (ax+b)^3 dx$.

FIVE MARK QUESTIONS

21. Solve $\int \frac{(x+1)^4}{x^2} dx$.

22. Solve $\int \frac{x+3}{x\sqrt{x}} dx$.

23. Solve $\int \left(\frac{3x^2+4x-5}{\sqrt{x}}\right) dx$.

24. Evaluate $\int \frac{1}{1+cosx} dx$.

25. Evaluate $\int sin^4 x \, dx$.

26. Solve $\int_1^2 \left(x^2 - 3x^{1/2} + \frac{1}{x^2}\right) dx$.

27. Solve $\int \frac{(x^2+4x)(2x-3)}{x^3} dx$.

28. Evaluate $\int \frac{1}{1+sinx} dx$.

29. Evaluate $\int cot^2 x \, dx$.

30. Evaluate $\int_0^{\frac{\pi}{6}} cos^2 \frac{x}{2} \, dx$.

TEN MARK QUESTIONS

31. Evaluate $\int \frac{x^2}{(a+bx)^3} dx$.

32. Solve $\int \frac{dx}{9x^2-4}$.

33. Solve $\int \frac{dx}{1-4x^2}$.

34. Evaluate $\int \frac{x^2}{1-x^6} dx$.

35. Solve $\int \frac{x^3+1}{x^2+1} \, dx$.

36. Solve $\int \frac{5x+1}{x^2-2x-35} \, dx$.

37. Solve $\int \frac{2x+1}{x^2+21x+3} \, dx$.

38. Solve $\int \frac{1}{(x+1)(x+2)}\,dx$.

39. Solve $\int \frac{dx}{(x+1)\sqrt{x^2+x+1}}$.

40. Solve $\int \frac{x^2}{\sqrt{x+5}}\,dx$.

UNIT – II

CHOOSE THE CORRECT ANSWER:

1.Find the value of $\int \cos(ax+b)dx$ is

a) $\frac{-1}{a}\sin(ax+b)$

b) $\frac{1}{a}\sin(ax+b)$

c) $\frac{-1}{a}\cos(ax+b)$

d) $\frac{1}{a}\cos(ax+b)$

2. Find the value of $\int \sec(ax+b)\tan(ax+b)dx$

a) $\frac{1}{a}\sin(ax+b)$

b) $\frac{1}{a}\cos(ax+b)$

c) $\frac{1}{a}\sec(ax+b)$

d) None of these

3.The value of $\int \frac{dx}{\sqrt{a^2-x^2}}$ is

a) $sin^{-1}\left(\frac{x}{a}\right)$

b) $cos^{-1}\left(\frac{x}{a}\right)$

c) $tan^{-1}\left(\frac{x}{a}\right)$

d) None of these

4. The value of $\int \frac{dx}{a^2+x^2}$ is

a) $\frac{1}{a} sin^{-1}\left(\frac{x}{a}\right)$

b) $\frac{1}{a} cos^{-1}\left(\frac{x}{a}\right)$

c) $\frac{1}{a} tan^{-1}\left(\frac{x}{a}\right)$

d) None of these

5. The solution of $\int \frac{dx}{4+9x^2}$

a) $\frac{1}{6} tan^{-1} \frac{3x}{2}$

b) $\frac{1}{8} tan^{-1} \frac{3x}{4}$

c) $\frac{1}{10} tan^{-1} \frac{3x}{6}$

d) $\frac{1}{4} tan^{-1} \frac{3x}{8}$

6.The solution of $\int \frac{dx}{\sqrt{4-9x^2}}$

a) $sin^{-1}\left(\frac{3x}{2}\right)$

b) $\frac{1}{3} sin^{-1}\left(\frac{3x}{2}\right)$

c) $\frac{1}{5} sin^{-1}\left(\frac{3x}{2}\right)$

d) $\frac{1}{7} sin^{-1}\left(\frac{3x}{2}\right)$

7. The solution of $\int \frac{dx}{x^2-a^2}$

a) $\frac{1}{2a} log\left(\frac{x-a}{x+a}\right)$

b) $\frac{1}{2a} log\left(\frac{x+a}{x-a}\right)$

c) $\frac{1}{2a} log\left(\frac{a-x}{a+x}\right)$

d) $\frac{1}{2a} log\left(\frac{a+x}{a-x}\right)$

8.If $\int \frac{dx}{1-4x^2}$. Find the A value is ?

a) 0

b) 2

c) 1

d) 3

9. Find the value of $\int \frac{x^2}{1+x^6} dx$ is

a) $tan^{-1}x$

b) $\frac{1}{3}tan^{-1}x^3$

c) $\frac{1}{5}tan^{-1}x^5$

d) $\frac{1}{7}tan^{-1}x^7$

10. Find the value of $\int \frac{e^x}{e^x+1}\ dx$ is

a) $\log e^x$

b) $\log e^x - 1$

c) $\log e^x + 1$

d) None of these.

ANSWERS:

1) b 2) c 3) a 4) c 5) a 6) b 7) a 8) c 9) b 10) c

TWO MARK QUESTIONS

11. Define Definite integral.

12. Define Indefinite integral.

13. Evaluate $\int (ax+b)^n dx$.

14. Evaluate $\int \sin(ax+b)dx$.

15. Evaluate $\int cosec^2(ax+b)dx$.

16. Evaluate $\int \frac{dx}{\sqrt{x^2-a^2}}$.

17. Evaluate $\int \frac{dx}{1+\frac{x^2}{4}}$.

18. Evaluate $\int \frac{1}{(3x+2)^2+16}dx$.

19. Evaluate $\int \frac{1}{(2-x)^2+16}dx$.

20. Evaluate $\int \frac{dx}{\sqrt{4-x^2}}$.

FIVE MARK QUESTIONS

21. Evaluate $\int \frac{dx}{\sqrt{a^2-x^2}}$.

22. Evaluate $\int \frac{1}{1+7x^2}dx$.

23. Evaluate $\int \frac{3x}{1+2x^4}dx$.

24. Solve $\int \frac{x^{24}}{x^{10}+1}dx$.

25. Solve $\int \frac{dx}{4x^2-4x+2}$.

26. Solve $\int \frac{2}{(1-x)(1+x^2)}dx$.

27. Solve $\int \frac{dx}{x(x^3+1)}$.

28. Solve $\int \frac{dx}{\sqrt{x(3-2x)}}$.

29. Prove that $\int_0^{\frac{\pi}{2}} \frac{(\sin x)^{\frac{3}{2}}}{(\sin x)^{\frac{3}{2}}+(\cos x)^{\frac{3}{2}}}\,dx = \frac{\pi}{4}$.

30. Reduction formula of $I_n = \int x^n e^{ax}\,dx$, where n is a positive integer.

TEN MARK QUESTIONS

31. Evaluate $\int \frac{1}{1-7x^2}\,dx$.

32. Evaluate $\int \frac{3x+5}{x^2+4x+7}\,dx$.

33. Solve $\int \sqrt{a^2-x^2}\,dx$.

34. Solve $\int (3x-2)\sqrt{x^2+x+1}\,dx$.

35. Evaluate $\int \sqrt{(x-3)(7-x)}\,dx$.

36.Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{9cosx+12\,sinx}$.

37. Reduction formula of $I_n = \int x^n cosax\,dx$.

38. Reduction formula of $I_n = \int cos^n x dx$.

39. Reduction formula of $I_n = \int sec^n x\,dx$.

40. Solve $\int \frac{6x+5}{\sqrt{6+x-2x^2}}\,dx$.

UNIT – III

CHOOSE THE CORRECT ANSWER:

1.Area bounded by the curves $y = x^2 + 2$, $y = -x$, $x = 0$ and $x = 1$ is

a) $\frac{17}{2}$

b) $\frac{17}{6}$

c) $\frac{19}{6}$

d) $\frac{13}{6}$

2. The area bounded between the curves $x = y^2$ and $x = 3 - 2y^2$ is

a) 2

b) 3

c) 4

d) 1

3. The area common to $y^2 = x$ and $x^2 = y$ is

a) 1

b) $\frac{2}{3}$

c) $\frac{1}{3}$

d) None of these

4. The area cut off from the parabola $4y = 3x^2$ by the straight line $2y = 3x + 12$ is

a) 25 sq. units

b) 27 sq. units

c) 36 sq. units

d) 16 sq. units

5. The area bounded by the curve $y = x^2 + 2x + 1$, the tangent at (1, 4) and the y axis is,

a) 1

b) $\frac{1}{2}$

c) $\frac{1}{3}$

d) $\frac{1}{4}$

6. Area enclosed by the curve $|x - 2| + |y + 1| = 1$ is

a) $\frac{2}{15}$ sq. units

b) $\frac{4}{15}$ sq. units

c) 2 sq. units

d) 4 sq. units

7. If the area bounded by a continuous function y = f(x), co ordinate axes and the line x = a, where a $\hat{I}R^+$, is equal to a e^a, then one such function can be

a) $e^x(x+1)$

b) $–e^x(x+1)$

c) e^x

d) None of these

8.Value of the parameter ‘a’ such that the area bounded by $y = a^2x^2 + ax + 1$, co ordinate axes and the line x = 1, attains at least value is,

a)$\frac{-1}{4}$

b) $\frac{-3}{4}$

c)$\frac{-1}{2}$

d) None of these

9. The slope of the tangent to a curve y = f(x) at (x, f(x)) is 2x +1. If the curve passes through the point (1, 2) then the area of the region bounded by the curve, the x axis and the line x = 1 is

a) $\frac{1}{6}$

b) 6

c) $\frac{5}{6}$

d)$\frac{6}{5}$

10. The area bounded by $y = |x - 1|$ and $y = 3 - |x|$ is

a) 2

b) 3

c) 4

d) 1

ANSWERS:

1) b 2) a 3) c 4) b 5) c 6) c 7) a 8) b 9) c 10) c.

TWO MARK QUESTIONS

11. Write the formula for area.

12. Define Area.

13. Write the formula for area in polar co – ordinates.

14. Write the formula for length of a curve.

15. Write the formula for length of the curve between two points (x_1, y_1) and (x_2, y_2).

16. Write the formula for polar co – ordinates.

17. Find the area of the loop of the curve $y^2 = x^4(x + 2)$.

18. Define area of a closed curve.

19. What is different between Cartesian co – ordinate and polar co – ordinate.

20. Find the area bounded by the curve $y = 3x - x^2$.

FIVE MARK QUESTIONS

21. Find the area bounded by one arch of the curve y = sin ax and the x axis.

22. Find the area bounded by one arch of the cycloid x = a (θ - sinθ), y = a(1 - cosθ) and its base.

23. Find the area bounded by the curve $x^2 = 4y$ the x axis and x = 2.

24. Find the entire area of the lamnicase of Bernoulli $r^2 = a^2 \cos 2\theta$.

25. Prove that the parameter of the cardioid r = a(1 + cosθ) is 8a.

26. Find the area of the ellipse $x^2 + 4y^2 – 6x + 8y + 9 = 0$.

27. Find the area bounded by $y = 5x – x^2 - 4$ and the x axis.

28. Find the area of the ellipse $3x^2 – 10xy + 10y^2 + 8x - 20y + 10 = 0$.

29. Find the area of the cardioids r = a(1 + cosθ).

30. Find the area enclosed within the curve r = 4 (1 + cosθ).

TEN MARK QUESTIONS

31. Find the area bounded by the curve $y^2 = 4ax$ the x axis and ordinate x = h.

32. Find the area of the loop of the curve $y^2 = x^2\left(\frac{a+x}{a-x}\right)$ the limits for the loop are –a and 0.

33. Find the area bounded by the parabola's $y^2 = 4ax$, $x^2 = 4by$.

34. Find the area enclosed between the parabola $y = x^2$ and the straight line $2x - y + 3 = 0$.

35. Find the length of the arc of the parabola $y^2 = 4ax$ from the vertex to any point (x, y) on the curve.

36. Find the area bounded by the x and y axes and the curve

(i) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

(ii) $\sqrt{x} + \sqrt{y} = 1$.

37. Find the area bounded by the curves $y = x^3$ and $y^2 = 9x$.

38. Prove that the area of the loop of the curve $y^2 (a + x) = x^2 (a - x)$ is $a^2(2 - \frac{\pi}{2})$.

39. Find the area of the parts into which the curve r = a(1 + cosθ) is divided by a line through the pole perpendicular to the initial line.

40. Find the area common to the two ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$.

UNIT – IV

CHOOSE THE CORRECT ANSWER:

1.In the region R let z = f(x, y) be a continuous function let us divide the region R into n parts as $\Delta s_1, \Delta s_2, \Delta s_3, \ldots \Delta s_n$ are called ___?

a) Sub region

b) Region

c) co –ordinate

d) None of these

2. The double integral over the region R of a function f(x, y) is

a) $\iint_R f(y,x)dx\,dy$

b) $\iint_R f(x,y)dx\,dy$

c) $\iint_R f(x,y)dy\,dx$

d) None of these

3. Find the value of $\int_0^1 \int_0^{x^2} (x^2+y^2)dydx$

a) $\frac{26}{103}$

b) $\frac{26}{101}$

c) $\frac{26}{105}$

d) $\frac{26}{107}$

4. Find the value of $\int_0^2 \int_0^4 xy\,dx\,dy$

a) 14

b) 15

c) 16

d) 17

5. Find the value of $\int_0^5 \int_0^2 (x+y)\,dx\,dy$

a) 15

b) 25

c) 35

d) 45

6. Find the value of $\int_1^2 \int_1^2 x^2y^2 \, dx \, dy$

a) $\frac{45}{9}$

b) $\frac{47}{9}$

c) $\frac{49}{9}$

d) $\frac{51}{9}$

7. Change the order of integration in $\int_0^a \int_x^a (x^2+y^2) \, dy \, dx$

a) $\frac{a^4}{3}$

b) $\frac{a^3}{3}$

c) $\frac{a^2}{3}$

d) None of these

8. Find the value of $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy \, dx$.

a) 0

b) 1

c) 2

d) 3

9. Find the value of $\int_0^a \int_0^x \int_0^y xyz \, dz \, dy \, dx$

a) $\frac{a^2}{48}$

b) $\frac{a^4}{48}$

c) $\frac{a^6}{48}$

d) None of these

10. Find the value of $\int_0^1 \int_0^1 \int_0^1 (xy + yz + zx)\, dx\, dy\, dz$

a) $\frac{1}{4}$

b) $\frac{3}{4}$

c) $\frac{5}{4}$

d) $\frac{7}{4}$

ANSWERS:

1) a 2) b 3) c 4) c 5) c 6) c 7) a 8) b 9) c 10) b

TWO MARK QUESTIONS

11. Evaluate $\int_0^2 \int_0^4 xy\, dx\, dy$.

12. Evaluate $\int_0^5 \int_0^2 (x + y) dx\, dy$.

13. Solve $\int_0^1 \int_0^1 dx\, dy$.

14. Evaluate $\int_0^a \int_0^b (x^2 + y^2) dx\, dy$.

15. Evaluate $\int_0^2 \int_1^2 xy\, dx\, dy$.

16. Solve $\int_1^2 \int_1^2 x^2 y^2 dx\, dy$.

17. Solve $\int_0^3 \int_0^2 \int_0^1 xyz\, dx\, dy\, dz$.

18. Define Double integral.

19. Define Sub region.

20.Write the formula for region of integration.

FIVE MARK QUESTIONS

21. Evaluate $\int\int_R (x^2 + y^2)\ dx\ dy$ where x is the region in the first quadrant for which $x + y \leq 1$.

22. Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \sin(\theta + \emptyset)\ d\theta\ d\emptyset$.

23. Solve $\int_0^3 \int_1^2 xy\ (x + y)\ dy\ dx$.

24. Evaluate $\iint x^2 + y^2\ dx\ dy$ over the area by the curves y = 4x, x + y = 3 and y = 0 to 2.

25. Solve $\int_0^1 \int_0^1 \int_0^1 (xy + yz + zx)\ dx\ dy\ dz$.

26.Evaluate $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^a r^2 sin\theta\ dr\ d\theta\ d\emptyset$.

27.Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{dz\ dy\ dx}{(x+y+z+1)^3}$.

28. $\int\int\int_R (x + y + z) dx\ dy\ dz$ where R is the region bounded by the planes x = 0, x =1 , y = 0, y = 1, z = 0 and z =1.

29.Evaluate $\int_0^a \int_0^{\sqrt{ax-x^2}} x^2\ dy\ dx$

30. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\ dz\ dy\ dx$.

TEN MARK QUESTIONS

31. Evaluate $\int\int xy\ dx\ dy$taken over the positive quadrant of the circle $x^2 + y^2 = a^2$.

32. Evaluate $\int\int (x + y)^2 dx\ dy$ over the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

33. Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} (x^2 + y^2) dy\ dx$.

34. Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} (x^2 + y^2) dy\ dx$ by changing in polar co – ordinates.

35. Evaluate $\int\int \frac{x^2y^2}{x^2+y^2}\ dx\ dy$ over the circular region between the circle $x^2 + y^2 = a^2$, $x^2 + y^2 = b^2$ (b > a) by transforming into polar co –ordinates.

36. Evaluate $\int \int \int_R (x-y+z)dx\,dy\,dz$ where R is given by $1 \le x \le 2$, $2 \le y \le 3$ and $1 \le z \le 3$.

37. Change the order of integration $\int_{-a}^{a} \int_{0}^{\sqrt{a^2-x^2}} x\,dx\,dy$.

38. Change the order of integration in $\int_0^a \int_x^a (x^2+y^2)dy\,dx$ and hence evaluate of it.

39. Evaluate $\int\int\int \frac{dx\,dy\,dz}{\sqrt{a^2-x^2-y^2-z^2}}$ the integral begin extendet to all positive values of the varieties of which the expansion is real.

40. Evaluate $\int_0^3 \int_0^2 \int_0^1 (x+y+z)dz\,dy\,dx$.

UNIT – V

CHOOSE THE CORRECT ANSWER:

1.The formula for $\Gamma(n+1)$ is

a) $\int_0^\infty x^n e^{-x}\,dx$

b) $\int_0^\infty x^n e^{x}\,dx$

c) $\int_0^\infty x^{-n} e^{-x}\,dx$

d) None of these

2. Find the value of $\Gamma\frac{7}{2}$

a) $\frac{2}{63}\Gamma\frac{9}{2}$

b) $\frac{4}{63}\Gamma\frac{11}{2}$

c) $\frac{6}{63}\Gamma\frac{13}{2}$

d) None of these

3. Find the value of $\int_0^\infty e^{-x^2}x^5 dx$

a) 0

b) 1

c) 2

d) 3

4. Find the value of $\int_0^\infty e^{-x^2} dx$

a) $\frac{1}{2}$

b) $\frac{\pi}{2}$

c) $\frac{\sqrt{\pi}}{2}$

d) $\frac{\sqrt{\pi}}{4}$

5. Find the value of $\int_{-\infty}^\infty e^{-x^2} dx$

a) π

b) $\sqrt{\pi}$

c) 1

d) 0

6. Find the value of $\int_0^1 \frac{dx}{\sqrt{log\frac{1}{x}}}$

a) π

b) $\sqrt{\pi}$

c) 1

d) 0

7. Find the value of $\int_0^{\frac{\pi}{2}} cos^6\theta \ d\theta$

a) $\frac{11\pi}{96}$

b) $\frac{13\pi}{96}$

c) $\frac{17\pi}{96}$

d) $\frac{15\pi}{96}$

8. Find the value of $\Gamma\frac{1}{2}$ is

a) π

b) $\sqrt{\pi}$

c) 1

d) 0

9. Find the value of $\Gamma\frac{-15}{2}$ is

a) $\frac{4}{195}\ \Gamma\frac{-11}{2}$

b) $\frac{2}{195}\ \Gamma\frac{-13}{2}$

c) $\frac{6}{195}\ \Gamma\frac{-15}{2}$

d) None of these

10. Find the value of $\int_0^{\frac{\pi}{2}}\sqrt{cot}\ \theta\ d\theta$ is

a) $\frac{1}{4}\Gamma\frac{1}{4}\Gamma\frac{3}{4}$

b) $\frac{1}{2}\Gamma\frac{1}{4}\Gamma\frac{3}{4}$

c) $\frac{1}{6}\Gamma\frac{1}{6}\Gamma\frac{3}{4}$

d) None of these

ANSWERS:

1) a 2) b 3) b 4) c 5) b 6) b 7) d 8) b 9) a 10) b

TWO MARK QUESTIONS

11. Define Gamma function.

12. Prove that $\Gamma 1 = 1$.

13. Define Beta function.

14. Prove that if n is a positive integer then, $\Gamma n + 1 = n!$

15. Prove that $\Gamma \frac{1}{2} = \sqrt{\pi}$.

16. Evaluate $\int_0^\infty e^{-x^2} x^5 dx$.

17. Evaluate $\int_0^\infty e^{-3x} x^4 dx$.

18. Write the formula for beta function interms of circular functions.

19. Prove that $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$.

20. Evaluate $\int_0^1 \frac{x^6}{\sqrt{1-x^2}}\, dx$.

FIVE MARK QUESTIONS

21. Prove that $\Gamma n + 1 = n\Gamma n$.

22. Evaluate $\Gamma \frac{-12}{10}$.

23. Prove that $\beta(m,n) = \beta\,(n,m)$.

24. Show that beta function in terms of circular function.

25. Show that beta function can be expressed as an infinite integral.

26. Show that $\int_0^1 \left[\log \frac{1}{y}\right]^{n-1} dy = \Gamma n$, n > 0.

27. Show that $\int_0^{\frac{\pi}{2}} \sqrt{\tan\theta}\; d\theta = \frac{\Gamma\frac{1}{4}\,\Gamma\frac{3}{4}}{2}$.

28. Evaluate $\int_0^1 \frac{dx}{\sqrt{1-x^4}}$.

29. Prove that $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$.

30. Evaluate $\int_0^{\frac{\pi}{2}} sin^p\theta\, cos^q\theta\; d\theta = \frac{\frac{1}{2}\Gamma\frac{p+1}{2}\Gamma\frac{q+1}{2}}{\Gamma\frac{p+q+2}{2}}$.

TEN MARK QUESTIONS

31. Show that $\int_0^\infty \sqrt{x}e^{-3\sqrt{x}}\ dx = \frac{315}{16}\sqrt{\pi}$.

32. Prove that $\beta(m,n) = \frac{\Gamma m\ \Gamma n}{\Gamma m+n}$.

33. Prove that $\int_0^\infty \int_0^\infty e^{-(ax^2+by^2)}x^{2m-1}y^{2n-1}\ dx\ dy = \frac{1}{4a^m b^n}\Gamma m\ \Gamma n$.

34. Express $\beta\left(n+\frac{1}{2}, n+\frac{1}{2}\right)$ in terms of gamma function in two different ways and hence prove that $\frac{\sqrt{\pi}\Gamma 2n+1}{2^{2n}\Gamma n+1}$.

35. Evaluate $\int_0^{\frac{\pi}{2}} sin^4\theta\ cos^6\theta\ d\theta$.

36. Evaluate $\int_0^{\frac{\pi}{2}} sin^{\frac{7}{2}}\theta\ cos^{\frac{5}{2}}\theta\ d\theta$.

37. Show that $\int_0^{\frac{\pi}{2}} \sqrt{sin\theta}\ d\theta . \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{sin\theta}}\ d\theta$.

38. Show that $\int_0^1 x^m(\log x)^n\ dx = \frac{(-1)^n\ n!}{(m+1)^{n+1}}$, n is an integer.

39. Prove that $\beta(m,\frac{1}{2}) = 2^{2m-1}\beta(m,m)$. Hence find $\Gamma(2m)$.

40. Express $\int_0^1 x^p(1-x^q)^m\ dx$ in terms of gamma function hence evaluate $\int_0^1 x^3\left(1-\sqrt{x}\right)dx$.