ஸ்ரீ-ல-ஸ்ரீ காசிவாசி சுவாமிநாத சுவாமிகள் கலைக் கல்லூரி

திருப்பனந்தாள் – 612504

S.K.S.S ARTS COLLEGE, THIRUPPANANDAL - 612504

QUESTION BANK

Title of the Paper

VECTOR CALCULUS AND FOURIER SERIES

Course: IIB.Sc (MATHS)

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CORE COURSE VII

VECTOR CALCULUS AND FOURIER SERIES

Objectives:

To provide the basic knowledge of vector differentiation & vector integration.
To solve vector differentiation & integration problems.

UNIT I

Vector differentiation –velocity & acceleration-Vector & scalar fields – Gradient of a vector- Directional derivative – divergence & curl of a vector solinoidal&irrotational vectors –Laplacian double operator –simple problems

UNIT II

Vector integration –Tangential line integral –Conservative force field –scalar potential- Work done by a force - Normal surface integral- Volume integral – simple problems.

UNIT III

Gauss Divergence Theorem – Stoke’s Theorem- Green’s Theorem – Simple problems & Verification of the theorems for simple problems.

UNIT IV

Fourier series- definition - Fourier Series expansion of periodic functions with Period 2and period 2a – Use of odd & even functions in Fourier Series.
□

UNIT V

Half-range Fourier Series – definition- Development in Cosine series & in Sine series Change of interval – Combination of series

TEXT BOOK(S)

1. M.L. Khanna, Vector Calculus, Jai PrakashNath and Co., 8th Edition, 1986.
2. S. Narayanan, T.K. ManicavachagamPillai, Calculus, Vol. III, S. ViswanathanPvt Limited, and Vijay Nicole Imprints Pvt Ltd, 2004.

UNIT – I - Chapter 1 Section 1 & Chapter 2 Sections 2.3 to 2.6 , 3 , 4 , 5 , 7 of [1]
UNIT – II - Chapter 3 Sections 1 , 2 , 4 of [1]
UNIT – III - Chapter 3 Sections 5 & 6 of [2]
UNIT – IV - Chapter 6 Section 1, 2, 3 of [2]
UNIT – V - Chapter 6 Section 4, 5.1, 5.2, 6, 7 of [2]

Reference:

1. P.Duraipandiyan and Lakshmi Duraipandian, Vector Analysis, Emarald publishers (1986).
2. Dr. S.Arumugam and prof. A.ThangapandiIssac, Fourier series, New Gamma

UNIT I

CHOOSE THE CORRECT ANSWER :

1. What is the Divergence of the vector field $\vec{f} = 3x^2\vec{\imath} + 5xy^2\vec{\jmath} + xyz^3\vec{k}$ at the point (1,2,3)
   a) 89
   b) 80
   c) 124
   d) 100

2. Divergence of $\vec{f}(x,y,z) = \frac{x\vec{\imath}+y\vec{\jmath}+z\vec{k}}{(x^2+y^2+z^2)^{3/2}}$
   a) 0
   b) 1
   c) 2
   d) 3

3. Curl of $\vec{f}(x,y,z) = 2xy\vec{\imath} + (x^2+z^2)\vec{\jmath} + 2zy\vec{k}$
   a) $xy^2\vec{\imath} - 2xyz\vec{k}$ and irrotational
   b) 0 and irrotational
   c) $xy^2\vec{\imath} - 2xyz\vec{k}$ and rotational
   d) 0 and rotational

4. Choose the curl of $\vec{f}(x,y,z) = x^2\vec{\imath} + xyz\vec{\jmath} - z\vec{k}$ at the point (2,1,2)
   a) $2\,\vec{\imath} + 2\vec{k}$
   b) $-2\,\vec{\imath} - 2\vec{\jmath}$
   c) $4\vec{\imath} - 4\vec{\jmath} + 2\vec{k}$
   d) $-2\,\vec{\imath} - 2\vec{k}$

5. Divergence of $\vec{f}(x,y,z) = e^{xy}\vec{\imath} - cosy\vec{\jmath} + (sinz)^2\vec{k}$
   a) $ye^{xy} + cosy + 2sinzcosz$
   b) $ye^{xy} - siny + 2sinzcosz$
   c) 0
   d) $ye^{xy} + siny + 2sinzcosz$

6. A vector field which has a finishing divergence is called as_____?
   a) Solenoidal
   b) Rotational
   c) Hemispherical
   d) Irrotational

7. Divergence of gradient of a vector function is equivalent to
    a) Laplacian operator
    b) Curl operator
    c) Double gradient
    d) Null vector

8. Divergence and curl of a vector field are ____?
    a) Scalar & scalar
    b) Scalar & vector
    c) Vector &vector
    d) Vector &scalar

9. A vector field with a vanishing curl is called as
    a) Irrotational
    b) Solenoidal
    c) Rotational
    d) Cycloidal

10. The curl of vector field $\vec{f}(x, y, z) = x^2\vec{\imath} + 2z\vec{\jmath} - y\vec{k}$ is _____?
    a) $-3\vec{\imath}$
    b) $-3\vec{\jmath}$
    c) $-3\vec{k}$
    d) 0

Answers: 1) b2) a 3) b 4) d 5) d 6) a 7) a 8) b 9) a 10) a

2 Marks :

11. If $\phi = x + xy^2 + yz^3$ to find $\nabla\phi$
12. Show that the vector $\vec{F} = x^2z^2\vec{\imath} + xyz^2\vec{\jmath} - xz^3\vec{k}$ isSolenoidal
13. If $\vec{r} = 5t^2\vec{\imath} + t\vec{\jmath} - t^3\vec{k}$ and $\vec{s} = sint\vec{\imath} - cost\vec{\jmath}$ find $\frac{d}{dt}(\vec{r}\,.\vec{s})$
14. Prove that curl(grad $\phi) = 0$
15. Show that $\vec{f} = yz\vec{\imath} + zx\vec{\jmath} + xy\vec{k}$ is irrotational
16. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1,1,1) in direction of $\vec{\imath} + \vec{\jmath} - \vec{k}$
17. Find grad r^n, where $r^2 = x^2 + y^2 + z^2$
18. If $\vec{A}$ and $\vec{B}$ are irrotational prove that $\vec{A} \times \vec{B}$ is solenoidal
19. Define vector point function
20. Define gradient

5 Marks :

21. Prove that $\frac{d}{dt}(\vec{A} \times \vec{B}) = \overrightarrow{A} \times \overrightarrow{\frac{dB}{dt}} + \overrightarrow{B} \times \overrightarrow{\frac{dA}{dt}}$

22. Find the directional derivative of $\phi = xy + yz + zx$ at (1,2,0) in the direction $\vec{\imath} + 2\vec{\jmath} + 2\vec{k}$, find also its maximum value
23. Prove that $\vec{A} = 3y^4z^2\vec{\imath} + 4x^3z^2\vec{\jmath} - 3x^2y^2\vec{k}$ is solenoidal
24. Show that if $\vec{A} = (6xy + z^3)\vec{\imath} + (3x^2 - z)\vec{\jmath} + (3xz^2 - y)\vec{k}$ is irrotational
25. Prove that $\nabla.(\phi\vec{F}) = \phi(\nabla.\vec{F}) + \vec{F}.(\nabla\phi)$
26. Prove that $\nabla^2(r^n\vec{r}) = n(n+3)r^{n-2}\vec{r}$
27. Findthe direction of magnitude of the maximum directional of $\phi = x^2yz^3$ at the point (2,1,-1)
28. If $\vec{a}$ is a constant vector and $\vec{r}$is the position vector of the point(x, y, z) with respect to the orgin, prove that
    i. Div$(\vec{a} \times \vec{r}) = 0$
    ii. Curl$(\vec{a} \times \vec{r}) = 2\vec{a}$
29. Find the directional derivative $\phi = x^2yz^3$ at (2,1,-1) in the direction $(-4\vec{\imath} - 4\vec{\jmath} + 12\vec{k})$
30. Prove that curl$(\vec{A} + \vec{B}) = curl(\vec{A}) + curl(\vec{B})$,where $\vec{A}$and $\vec{B}$ are differentiable vector function.

10 Marks:

31. Show that $\vec{F} = (y^2 - z^2 + 3yz - 2x)\vec{\imath} + (3xz + 2xy)\vec{\jmath} + (3xy - 2xz + 2z)\vec{k}$ is both solenoidal and irrotational
32. a) If $\vec{A} = x^2y\vec{\imath} - 2xz\vec{\jmath} + 2yz\vec{k}$, find curl curl $\vec{A}$
    b) Prove that $\nabla \times (\phi\vec{A}) = \phi(\nabla \times \vec{A}) + (\nabla\phi) \times \vec{A}$
33. If $\phi = x^2y^3z^4$find div(grad ϕ) and curl (grad ϕ)
34. Find $\nabla^2(r^n)$ and deduce that $\nabla^2(1/r)$ where $r = |\vec{r}| = |x\vec{\imath} + y\vec{\jmath} + z\vec{k}|$
35. Find the equation of the tangent plane and the normal to the surface $xyz = 4$ at the point (1,2,2) on it
36. Find div$\vec{F}$ and curl $\vec{F}$,where $\vec{F} = x^2y\vec{\imath} - (z^3 - 3x)\vec{\jmath} + 4y^2\vec{k}$
37. If $\vec{A} = 2x^2\vec{\imath} - 3yz\vec{\jmath} + xz^2\vec{k}$ and $\phi = (2z - x^3y)$ and find $\vec{A}.\nabla\phi$ and $\vec{A} \times \nabla\phi$ at (1,-1,1)
38. Find $\nabla.\vec{F}$ and $\nabla \times \vec{F}$ at (1,-1,1) if $\vec{F} = xz^3\vec{\imath} - 2x^2yz\vec{\jmath} + 2yz^4\vec{k}$
39. If curl $\vec{F}$ is zero , find the value of a ,when
$$\vec{F} = (axy - z^3)\vec{\imath} + (a - z)x^2\vec{\jmath} + (1 - a)xz^2\vec{k}$$
40. If $\vec{A} = t^2\vec{\imath} - t\vec{\jmath} + (2t + 1)\vec{k}$and $\vec{B} = (2t - 3)\vec{\imath} - \vec{\jmath} - t\vec{k}$ to find $i)$ $\frac{d}{dt}(\vec{A}.\vec{B})$
    ii) $\frac{d}{dt}(\vec{A} \times \vec{B})$iii) $\frac{d}{dt}(\vec{A}.\vec{A})$ at $t = 1$.

UNIT II

CHOOSE THE CORRECT ANSWER:

1. Line integral is used to calculate
   a) Force
   b) Area
   c) Volume
   d) Length

2. Surface integral is used to compute
   a) Surface
   b) Area
   c) Volume
   d) Density

3. A field in which a test charge around any closed surface in static path is zero is called
   a) Solenoidal
   b) Rotational
   c) Irrotational
   d) Conservative

4. Calculate the area enclosed by parabolas $x^2 = y$ and $y^2 = x$
   a) ½
   b) $\frac{1}{3}$
   c) ¼
   d) $\frac{1}{6}$

5. Evaluate $\int_0^\infty \int_0^{\frac{\pi}{2}} e^{-r^2} r d\theta dr$
   a) π
   b) $\frac{\pi}{2}$
   c) $\frac{\pi}{4}$
   d) $\frac{\pi}{8}$

6. Evaluate $\int xydx\,dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$
   a) $a^4/8$
   b) $a^4/4$
   c) $a^2/8$
   d) $a^2/4$

7. To find volume ____________ can be used
   a) Single integration
   b) Double integration
   c) Triple integration
   d) Double and triple integration

8. Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)dx\,dy\,dz$
   a) 0
   b) 1
   c) 2
   d) 3

9. What is the volume of a cube with side a?
   a) $a^3/8$
   b) a^2
   c) a^3
   d) $a^{2/}4$

10. Find the value of $\int_{0}^{1}\int_{0}^{2}\int_{1}^{2} x\,y^2 z^3 dx\,dy\,dz$
    a) 2
    b) 3
    c) 4
    d) 5

Answers : 1) d 2) b 3) d 4) b 5) c 6) a 7) a 8) a 9) c 10) d

2 Marks :

11. Define conservative field
12. Define surface integral of a vector point function
13. Find $\iiint \nabla.\vec{r}\,dv$
14. Find $\int_c \overrightarrow{dr}$ where c is the curve whose parameteric equations are $x = t$, $y = t^2, z = t^3$drawn from (0,0,0) to (1,1,1,)
15. If $\vec{F} = x^2\vec{\imath} + y^2\vec{\jmath}$, evaluate $\int_c \vec{F}.\overrightarrow{dr}$ along the line $y = x$ from (0,0) to (1,1)
16. Show that $\vec{F} = x^2\vec{\imath} + y^2\vec{\jmath} + z^2\vec{k}$ is a conservative vector field.
17. Define work done by a force
18. Evaluate $\iint_s \vec{r}.\hat{n}ds$, where s is a closed surface.
19. Define line integral
20. Show that $\iint_s curl\,\vec{F}.\hat{n}\,ds = 0$ where s is any closed surface.

5 Marks :

21. If $\vec{F} = x^2\vec{\imath} + xy\vec{\jmath}$ evaluate $\int_c \vec{F}.\overrightarrow{dr}$ from (0,0) to (1,1) along the line $y = x$
22. Given the vector field $\vec{F} = xz\vec{\imath} + yz\vec{\jmath} + z^2\vec{k}$ evaluate $\int_c \vec{F}.\overrightarrow{dr}$ from the point (0,0,0) to (1,1,1) where c is the closed curve and $x = t\,, y = t^2, z = t^3$
23. Evaluate $\int_c (xdy - ydx)$around the circle $x^2 + y^2 = 1$
24. If $\vec{F} = (3x^2 + 6y)\vec{\imath} - 14yz\vec{\jmath} + 20xz^2\vec{k}$ and evaluate $\int_c \vec{F}.\overrightarrow{dr}$ where c is the straight line joining (0,0,1) to (1,1,1)
25. Find the total work done in moving a particle of a force field given by $\vec{F} = (2x - y + z)\vec{\imath} + (x + y - z)\vec{\jmath} + (3x - 2y - 5z)\vec{k}$along a circle c in the xy plane $x^2 + y^2 = 9, z = 0$
26. If $\vec{A} = (x - y)\vec{\imath} + (x + y)\vec{\jmath}$, evaluate $\int_c \vec{A}.\overrightarrow{dr}$ around the closed curve c which is given by the equation $x^2 = y$ and $y^2 = x$
27. Evaluate $\iiint_V \nabla.\vec{F}dv$if $\vec{F} = x^2\vec{\imath} + y^2\vec{\jmath} + z^2\vec{k}$ and if v is the volume of the region enclosed by the cube $0 \le x, y, z \le 1$
28. Evaluate $\int_c \vec{F}.\overrightarrow{dr}$where $\vec{F} = x^2\vec{\imath} + y^3\vec{\jmath}$ and c is the portion of the parabola $y = x^2$ in the xy plane from A(0,0) to B(1,1)
29. Find the circulation of$\vec{F}$round the curve c where$\vec{F} = y\vec{\imath} + z\vec{\jmath} + x\vec{k}$ and c is the curve of $x^2 + y^2 = 1, z = 0$
30. If $\vec{F} = 2xz\vec{\imath} - x\vec{\jmath} + y^2\vec{k}$ then evaluate$\iiint_V \vec{F}.dv$ where v is the region bounded by the surface $x = 0, x = 2, y = 0\ to\ 6$ and $z = x^2\ to\ 4$.

10 Marks:

31. Evaluate $\iint_s \vec{A}.\hat{n}\,ds$ if $\vec{A} = (x + y^2)\vec{\imath} - 2x\vec{\jmath} + 2yz\vec{k}$ and s is the surface of the plane $2x + y + 2z = 6$ in the first octant
32. $\iint_s \vec{A}.ds$ to find ,where $\vec{A} = 12x^2y\vec{\imath} - 3yz\vec{\jmath} + 2z\vec{k}$ where s is the portion of the plane $x + y + z = 1$ in the first octant
33. Show that $\iint_s \left(yz\vec{\imath} + zx\vec{\jmath} + xy\vec{k}\right).\hat{n}\,ds = \frac{3}{8}$ where s is the surface of the sphere $x^2 + y^2 + z^2 = 1$, in the first octant.
34. Evaluate $\int_v div\vec{A}dv$, where $\vec{A} = 2x^2y\vec{\imath} - y^2\vec{\jmath} + 4xz^2\vec{k}$ and v is the region in the first octant bounded by the cylinder,$y^2 + z^2 = 9, x = 2$
35. Evaluate $\int_c \vec{F}.\overrightarrow{dr}$, where $\vec{F} = (x^2 + y^2)\vec{\imath} - 2xy\vec{\jmath}$, c is the curve the rectangle in the xy plane bounded by $y = 0, x = a, y = b$ and $x = 0$
36. Evaluate $\iint_s \vec{F}.\hat{n}\,ds$ where $\vec{F} = z\vec{\imath} + x\vec{\jmath} - y^2z\vec{k}$ and s is surface integral of the cyclinder $x^2 + y^2 = 1$ included in the first octant between the planes $z = 0$ and$z = 2$
37. If $\vec{F} = (2x^2 - 3z)\vec{\imath} - 2xy\vec{\jmath} - 4x\vec{k}$ then evaluate i) $\iiint_V \nabla \times \vec{F}dv$

ii) $\iiint_V \nabla.\vec{F}dv$,where v is the region bounded by $x = 0, y = 0$ $z = 0$ and $2x + 2y + z = 4$

38. Evaluate $\iint_s \vec{F}.\hat{n}\,ds$, where $\vec{F} = 18z\vec{i} - 12\vec{j} + 3y\vec{k}$ as s is the part of the plane $2x + 3y + 6z = 12$which is in the first octant

39. Evaluate $\iiint_V \vec{F}.dv$, where $\vec{F} = xy\vec{i} - zx\vec{j} + \vec{k}$ and v is the volume of the sphere $x^2 + y^2 + z^2 = 4$ and $x \geq 0, y \geq 0$ and $z \geq 0$

40. a) If $\vec{F} = 3xy\vec{i} - y^3\vec{j}$ then evaluate $\int_c \vec{F}.\overrightarrow{dr}$ where c is the circle of $y = 2x^2$ from A(0,1) and B (0,2)

    b) Evaluate $\int_c \vec{F}.\overrightarrow{dr}$ where $\vec{F} = 3x^2\vec{i} + (2xz - y)\vec{j} + z\vec{k}$ and c is the straight line from A(0,0,0) to B(2,1,3)

UNIT III

CHOOSE THE CORRECT ANSWER :

1. Find the value of Stoke's theorem for $y\vec{i} + z\vec{j} + x\vec{k}$
   a) $\vec{i} + \vec{j}$
   b) $\vec{j} + \vec{k}$
   c) $\vec{i} + \vec{j} + \vec{k}$
   d) $-\vec{i} - \vec{j} - \vec{k}$

2. The Stoke's theorem uses which of the following operation?
   a) Divergence
   b) Gradient
   c) Curl
   d) Laplacian

3. Which of the following theorem convert line integral to surface integral?
   a) Gauss divergence and Stoke's theorem
   b) Stoke's and greens theorem
   c) Stoke's theorem only
   d) Green's theorem only

4. Find the value of divergence theorem for $D = 2xy\vec{i} + x^2\vec{j}$ for the rectangular parallelepiped given by $x = 0$ and 1, $y = 0$ and 2, $z = 0$ and 3
   a) 10
   b) 12
   c) 14
   d) 16

5. If a function is said to be harmonic,then
   a) Curl (Grad v)=0
   b) Div (curl v)=0
   c) Div (grad v)=0
   d) Grad (curl v)=0

6. Mathematically, the function is Green's theorem will be
    a) Continuous derivatives
    b) Discrete derivatives
    c) Continuous partial derivatives
    d) Discrete partial derivatives
7. Find the value of Green's theorem for $F = x^2$ and $G = y^2$ is
    a) 0
    b) 1
    c) 2
    d) 3

8. The path traversal in calculating the Green's theorem is
    a) Clockwise
    b) Anticlockwise
    c) Inwards
    d) Outwards

9. Application of Green's theorem are meant to be
    a) One dimensional
    b) Two dimensional
    c) Three dimensional
    d) Four dimensional

10. Gauss divergence theorem converts
    a) Line to surface integral
    b) Line to volume integral
    c) Surface to line integral
    d) Surface to volume integral

Answers : 1) d 2) c 3) b 4) b 5) c 6) c 7) a 8) b 9) b 10) d

2 Marks :

11. State Gauss divergence theorem
12. State Stoke's theorem
13. Show that $\iint_s \vec{r}\,.\overrightarrow{ds} = 3V$, where $\vec{r} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$, s is a closed surface enclosing volume V.
14. If $\hat{n}$ is the unit outward drawn normal to any close surface of area S, show that $\iiint div\,\hat{n}\,dv = S$
15. Prove that $\int \phi\nabla\phi\overrightarrow{dr} = 0$
16. Use divergence theorem, evaluate $\iint_s \nabla r^2.\hat{n}\overrightarrow{ds} = 0$
17. UsingStoke's theorem prove that $\int \vec{r}\,.\overrightarrow{dr} = 0$

18. If $\vec{f} = curl\,\vec{A}(\nabla \times \vec{A})$ prove that $\iint_s \vec{F}.\hat{n}\,ds = 0$ for any closed surface
19. Evaluate by Stoke's theorem $\int_c (e^x dx + 2y\,dy - dz)$where c is the curve $x^2 + y^2 = 4, z = 2$
20. State green's theorem

5 Marks :

21. If $\vec{F} = curl\,\vec{A}$, prove that $\iint_s \vec{F}.\hat{n}\,ds = 0$ for any closed surface S.
22. Evaluate $\oint_c y(2xy - 1)dx + x(2xy + 1)dy$,where c is the circle $x^2 + y^2 = 1$ using green's theorem.
23. Using divergence theorem show that $\iint_s (ax\vec{\imath} + by\vec{\jmath} + cz\vec{k}).\hat{n}\,ds = \frac{4}{3}\pi(a + b + c)$, where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$
24. Prove that area of elipse using Green's theorem
25. Evaluate $\iint_s \vec{F}.\hat{n}\,ds$ use divergence theorem for $\vec{F} = x^2\vec{\imath} + y^2\vec{\jmath} + z^2\vec{k}$,where S is the surface of the cubic formed by the planes $x = 0, x = a$ and $y = 0\ to\ b, z = 0\ to\ c$
26. Use Green's theorem in a plane to evaluate $\int_c (2x - y)dx + (x + y)dy$, where C is the bounbdary of the circle $x^2 + y^2 = a^2$ in the x and y plane
27. Use Green's theorem to evaluate $\int_c yx^2dx - x^2dy$, where C is the half of the circle, $x^2 + y^2 = 25$
28. Use Stoke's theorem to evaluate $\iint_s curl\vec{F}.\overrightarrow{ds}$ where $\vec{F} = z^2\vec{\imath} - 3xy\vec{\jmath} + x^3y^3\vec{k}$and s is the part of $z = 5 - x^2 - y^2$ above the plane $z = 1$
29. Evaluate $\iint_s \vec{F}.\hat{n}\,ds$, where $\vec{F} = x^3\vec{\imath} + y^3\vec{\jmath} + z^3\vec{k}$ and s is a surface of the sphere $x^2 + y^2 + z^2 = a^2$
30. Evaluate $\int_c \vec{F}.\overrightarrow{dr}$by Stoke's theorem where $\vec{F} = y^2\vec{\imath} + x^2\vec{\jmath} - (x + 2)\vec{k}$ and c is the boundary of the triangle with vertices at (0,0,0),(1,0,0) and (1,1,0)

10 Marks:

31. Verify divergence theorem, where $\vec{F} = (x^2 - yz)\vec{\imath} + (y^2 - zx)\vec{\jmath} + (z^2 - xy)\vec{k}$ taken over the rectangular parallel $0 \le x \le a,\ 0 \le y \le b, 0 \le z \le c$
32. Verify gauss divergence for the function $\vec{A} = 2x^2y\vec{\imath} - y^2\vec{\jmath} + 4x^2z\vec{k}$ taken over the region bounded by the cyclinder $y^2 + z^2 = 9$ and the planes $x = 0$, $x = 2, y = 0, z = 0$lying in the first octant

33. Verify Green's theorem in the plane for $\oint_c (3x^2 - 8y^2)\, dx + (4y - 6xy)dy,$ where c is the boundary given by $y = \sqrt{x}$ and $y = x^2$.
34. Verify Green's theorem in the plane $\int_c (xy + y^2)\, dx + x^2dy,$ where c is the curve enclosing the region are bounded the parabola $y = x^2$ and the line $y = x$
35. Verify Stoke's theorem $\vec{F} = y\vec{\imath} + z\vec{\jmath} + x\vec{k}$, where s is the upper half of the surface of the sphere $x^2 + y^2 + z^2 = 1$ and c is the boundary of the curve
36. Verify stoke's theorem $\vec{A} = (2x - y)\vec{\imath} + yz^2\vec{\jmath} - y^2z\vec{k}$ taken over the half of the surface the sphere $x^2 + y^2 + z^2 = 1, z \geq 0$
37. Verify gauss divergence for $\vec{F} = (4xz)\vec{\imath} - y^2\vec{\jmath} + yz\vec{k}$ taken over the cube bounded by $x = 0, x = 1\,, y = 0\,, y = 1\; and\; z = 0, z = 1$
38. Verify divergence theorem,$\vec{F} = 4x\vec{\imath} - 2y^2\vec{\jmath} + z^2\vec{k}$ taken over by the region bounded by $x^2 + y^2 = 4, z = 0\; and\; dz = 3$
39. Evaluate the integral $\int_c (x + y)\, dx + (2x - z)dy + (y + z)dz$ where c is the boundary of the triangle vertices (2,0,0). (0,3,0) and (0,0,6) using stoke's theorem
40. Evaluate $\int_c xy\, dx + xy^2dy$ by stokes theorem, where c is the square of the xy plane with vertices (1,0), (-1,0) and (0,1), (0,-1)

UNIT IV

CHOOSE THE CORRECT ANSWER :

1. Which of the following is an " even" function of t?
   a) t^2
   b) $t^2 - 4t$
   c) $\sin(2t) + 3t$
   d) $t^3 + 6$

2. A "periodic function" is given by a function which
   a) Has a period $T = 2\pi$
   b) Satisfies $f(t + T) = f(t)$
   c) Satisfies $f(t + T) = -f(t)$
   d) Has a period $T = \pi$

3. The Fourier series of an odd periodic function,contains only
   a) Cosine terms
   b) Sine terms
   c) Odd harmonics
   d) Even harmonics

4. The trigonometric Fourier series of an even function of time does not have
   a) Sine terms
   b) Odd terms
   c) Cosine terms
   d) None of these

5. Trigonometric Fourier series of a periodic function only have
   a) Cosine and sine terms
   b) Sine terms
   c) Cosine terms
   d) None of these

6. If the function $f(x)$ is even , then which of the following is zero?
   a) a_n
   b) b_n
   c) a_0
   d) Nothing is zero

7. If the function $f(x)$ is odd, then which of the following is only coefficient is present ?
   a) a_n
   b) b_n
   c) a_0
   d) Everything is present

8. Find a_n if the function $f(x) = x - x^3$
   a) Finite value
   b) Infinite value
   c) Zero
   d) Can't be found

9. Find b_n if the function $f(x) = x^2$
   a) Finite value
   b) Infinite value
   c) Zero
   d) Can't be found

10. Find a_0 of the function $f(x) = \frac{1}{4}(\pi - x)^2$
    a) $\frac{\pi^2}{6}$
    b) $\frac{\pi^2}{12}$
    c) $\frac{5\pi^2}{6}$
    d) $\frac{5\pi^2}{12}$

Answers: 1) a 2) b 3) b 4) a 5) c 6) b 7) b 8) c 9) c 10) a

2 Marks :

11. Define odd and even functions
12. Find the constant terms in the expansion of $f(x) = \frac{1}{2}(\pi - x), 0 < x < 2\pi$
13. Define Fourier series
14. Write the properties of odd and even functions
15. Define periodic function
16. Show that a constant has any positive number as period
17. Find b_n in the expansion of x^2 as a fourier series in $(-\pi, \pi)$
18. Determine the value of a_n in the fourier series expansion of $f(x) = x^3$ in $-\pi < x < \pi$
19. Classify the function as even or odd $f(x) = \begin{cases} sinx, & 0 < x < \pi \\ -sinx, & -\pi < x < 0 \end{cases}$
20. Determine the Fourier series for the function $f(x) = x^2$ of period 2π in $0 < x < 2\pi$, find a_0

5 Marks:

21. Express $f(x) = x$ for $-\pi < x < \pi$ as a fourier series with period 2π
22. If $f(x) = -x$ in $-\pi < x < 0$ as the interval of fourier series , deduce that
$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$
23. Find the Fourier series of the function $f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 \le x \le \pi \end{cases}$
24. If $f(x) = \frac{1}{2}(\pi - x)$ for $-\pi < x < \pi$ as a fourier series with period 2π in the interval (0, 2π)
25. Obtain the Fourier series for the function $f(x) = \begin{cases} 1, & 0 < x < \pi \\ 0, & \pi < x < 2\pi \end{cases}$
26. Find the Fourier series for $f(x) = x^2$ in $-\pi \le x \le \pi$ and deduce that
$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$$
27. Find the Fourier expansion of $f(x) = x$ in $-\pi < x < \pi$
28. Expand the function $f(x) = x^2$ as Fourier series in $(-\pi, \pi)$ hence deduce that
$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$$
29. Find the Fourier series for the periodic 2π and defined on the interval $(-\pi, \pi)$
$$f(x) = \begin{cases} 0, & if\ -\pi \le x \le 0 \\ 1, & if\ \ 0 < x \le \pi \end{cases}$$
30. Obtain the Fourier series $f(x) = |x|$ if $-\pi < x < \pi$

10 Marks :

31. Find the Fourier series expansion for $f(x) = \begin{cases} -\pi\,, & \pi < x < 0 \\ x\,, & 0 < x < \pi \end{cases}$ and deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$
32. If $f(x) = sinx$ for 0 to 2π in Fourier series with period 2π
33. Express $f(x)$ as the Fourier series in the range (0, 2π) for $f(x) = \begin{cases} x\,, & 0 < x < \pi \\ 2\pi - x\,, & \pi < x < 2\pi \end{cases}$
34. Expand $f(x) = x(2\pi - x)$ as fourier series in (0, 2π) and hence deduce that the sum of $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$
35. Expand the Fourier series of $f(x) = x\,sinx$ for $0 < x < 2\pi$ and deduce that
$$\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \cdots = \frac{\pi - 2}{4}$$
36. Express $f(x) = xsinx$ as a Fourier series in $(-\pi, \pi)$
37. Expand $f(x) = |cosx|$ in a Fourier series in the interval $(-\pi, \pi)$
38. Obtain the Fourier series $f(x) = |x|$, if $-\pi < x < \pi$ and hence deduce that
$$\frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{5^2} \ldots = \frac{\pi^2}{8}$$
39. If $f(x) = x$ and range $0 < x < 2\pi$ in Fourier series with period 2π.
40. Express $f(x) = \pi^2 - x^2$ for $-\pi < x < \pi$ as a Fourier series with period 2π.

UNIT V

CHOOSE THE CORRECT ANSWER :

1. In half range cosine Fourier series, we assume that the function to be
   a) Odd function
   b) Even function
   c) Can't determined
   d) Can be anything

2. Find the half sine series of the function $f(x) = x$, when $0 < x < \frac{\pi}{2}$ and $(\pi - x)$ when $\frac{\pi}{2} < x < \pi$
   a) $8/\pi\left[\frac{sinx}{1^2} - \frac{\sin(3x)}{3^2} + \frac{\sin(5x)}{5^2} - \frac{\sin(7x)}{7^2} + \cdots\right]$
   b) $4/\pi\left[\frac{sinx}{1^2} + \frac{\sin(3x)}{3^2} + \frac{\sin(5x)}{5^2} + \frac{\sin(7x)}{7^2} + \cdots\right]$
   c) $8/\pi\left[\frac{sinx}{1^2} + \frac{\sin(3x)}{3^2} + \frac{\sin(5x)}{5^2} + \frac{\sin(7x)}{7^2} + \cdots\right]$
   d) $4/\pi\left[\frac{sinx}{1^2} - \frac{\sin(3x)}{3^2} + \frac{\sin(5x)}{5^2} - \frac{\sin(7x)}{7^2} + \cdots\right]$

3. Find b_n when we have to find the half range sine series of the function x^2 in the interval 0 to 3

a) $-\frac{18\cos(n\pi)}{n\pi}$
b) $\frac{18\cos(n\pi)}{n\pi}$
c) $\frac{-18\cos(n\pi/2)}{n\pi}$
d) $\frac{18\cos(n\pi/2)}{n\pi}$

4. What is the value of a_0 if the function is $f(x) = x^3$in the interval 0 to 5?
   a) 25/4
   b) 125/4
   c) 625/4
   d) 5/4

5. Find the value of $1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$ when finding the half range Fourier sine series of the function $f(x) = 1$ in $0 < x < \pi$
   a) $\frac{\pi^2}{4}$
   b) $\frac{\pi^2}{8}$
   c) $\frac{\pi^2}{2}$
   d) $\frac{3\pi^2}{8}$

6. Find the value of $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \cdots$by finding the fourier cosine series of the function $f(x) = x$ in the interval $0 < x < 1$
   a) $\frac{\pi^4}{12}$
   b) $\frac{\pi^4}{48}$
   c) $\frac{\pi^4}{24}$
   d) $\frac{\pi^4}{96}$

7. Find a_n if the function $f(x) = x - x^3$
   a) Finite value
   b) Infinite value
   c) Zero
   d) Can't be found

8. Find $b_n$if the function $f(x) = x^2$
   a) Finite value
   b) Infinite value
   c) Zero
   d) None of these

9. If the function $f(x)$ is odd, then which of the only coefficient is present ?
   a) a_n

b) b_n
c) a_0
d) Everything is present

10. Find a_0 of the function $f(x) = \frac{1}{4}(\pi - x)^2$
a) $\frac{\pi^2}{6}$
b) $\frac{\pi^2}{12}$
c) $\frac{5\pi^2}{6}$
d) $\frac{5\pi^2}{12}$

Answers: 1) b 2) d 3) a 4) c 5) b 6) d 7) c 8) c 9) b 10) a

2 Marks:

11. Write the formula for Fourier constants to expand $f(x)$ as a sine series in $(0, \pi)$
12. Find the Fourier sine series of $f(x) = x$ in $0 < x < 2$
13. Expand $f(x) = 1$ in a sine series $0 < x < \pi$
14. Find a_0 in the cosine series of $f(x) = x$ in $0 < x < 2$
15. Find a Fourier sine series of $f(x) = ax + b$ in $0 < x < 1$
16. Write the formula for cosine series
17. Find a_0 in the cosine series $f(x) = \pi - x$ in the range $0 \leq x \leq \pi$
18. Find the sine series of $f(x) = e^x$in (0, π)
19. Find the Cosine series of $f(x) = e^x$ in (0, 1)
20. Find the Fourier cosine series of $f(x) = cos^2x, 0 < x < \pi$

5 Marks:

21. Expand the function $f(x) = x$, $0 < x < \pi$ in the Fourier sine series
22. Find half range sine series for the function $f(x) = x - x^2, 0 < x < 1$
23. Find the sine series of the function $f(x) = x^2$ in (0, π)
24. Obtain the cosine series for $f(x) = \begin{cases} cosx & , 0 < x < \pi/2 \\ 0 & , \frac{\pi}{2} < x < \pi \end{cases}$
25. Find a sine series for $f(x) = c$ in the range $0\ to\ \pi$
26. Find the cosine series corresponding to the function $f(x) = x,$ defined the interval $0\ to\ \pi$
27. Find the half range cosine series of $f(x) = \pi - x^2$ in the interval (0,π)
28. Find the cosine series for $f(x) = x^2$ in $0 < x < \pi$
29. Expand the sine series of the function $f(x) = x\ sinx$ in $0 < x < \pi$
30. Find the cosine series for $f(x) = (x - 1)^2$ in (0,1)

10 Marks:

31. Find the cosine series in the range $0 \leq x \leq \pi$ for $f(x) = \pi - x$
32. Show that $x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n cosnx \,/\, n^2$ in the interval $-\pi \leq x \leq \pi$ deduce that
    i. $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots = \frac{\pi^2}{12}$
    ii. $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$
    iii. $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$
33. Show that in the range 0 to 2π the expansion of e^x as a Fourier series that in the $e^x = \frac{e^{2\pi}-1}{\pi}\left[\frac{1}{2} + \sum_{n=1}^{\infty}\frac{\cos nx}{n^2+1} - \frac{n \sin nx}{n^2+1}\right]$
34. If the function $y = x$ in the range 0 to π is expanded as the sine series , show that is equal to $2sinx - \frac{sin2x}{2} + \frac{sin3x}{3} - \cdots$
35. Find the sine series for $f(x) = x(\pi - x)$ in $(0, \pi)$ deduce that $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \cdots$
36. Find the sine series of $f(x) = \begin{cases} x & , 0 \leq x \leq \frac{\pi}{2} \\ (\pi - x) , & \frac{\pi}{2} \leq x \leq \pi \end{cases}$
37. Expand the function $f(x) = sinx ,\ 0 \leq x \leq \pi$ is an cosine series
38. Find the Fourier cosine series for $f(x) = xsinx$ in the range $0 < x < \pi$
39. If $f(x) = \begin{cases} sinx , & 0 \leq x \leq \pi \\ 0 , & \pi \leq x \leq 2\pi \end{cases}$ find a Fourier series of periodic 2π and hence evaluate $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \cdots$
40. Find the cosine series for the function $f(x) = x(\pi - x)$ in the range $0 < x < \pi$