ஸ்ரீ-ல-ஸ்ரீ காசிவாசி சுவாமிநாத சுவாமிகள் கலைக் கல்லூரி

திருப்பனந்தாள் – 612504

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QUESTION BANK

Title of the Paper

ABSTRACT ALGEBRA

Course: IIIB.Sc (Maths)

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CORE COURSE XII
ABSTRACT ALGEBRA

Objectives

1. To introduce the concept of Algebra from the basic set theory and Functions, etc.
2. To introduce the concept of Group theory and Rings.

UNIT I

Groups : Definition and Examples – Elementary Properties of a Group – Equivalent Definitions of a Group.-Permutation Groups

UNIT II

Subgroups - Cyclic Groups-Order of an Element – Cosets and Lagrange's Theorem .

UNIT III

Normal Subgroups and Quotient Groups - Isomorphism –Homomorphism

UNIT IV

Rings: Definitions and Examples - Elementary properties of rings – Isomorphism - Types of rings.-Characteristic of a ring – subrings – Ideals - Quotient rings

UNIT V

Maxi mal and Prime Ideals.-Homomorphism of rings – Field of quotient of an integral domain – unique factorization domain-Euclidean domain

Textbook

1. S Arumugam and AThangapandi Isaac, Modern Algebra, SciTech Publications, Chennai, 2003.

Unit 1: Chapter 3 Sections 3.1-3.4
Unit 2: Chapter 3 Sections3.5-3.8
Unit 3: Chapter 3 Sections 3.9-3.11
Unit 4: Chapter 4 Sections 4.1-4.8
Unit 5: Chapter 4 Sections 4.9- 4.11, 4.13-14

References

1. N. Herstein, Topics in Algebra, John Wiley & Sons, Student 2nd edition, 1975.
2. Vijay, K. Khanna and S.K. Bhambri, A Course in Abstract Algebra, Vikas Publishing House Pvt. Ltd.

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UNIT – I

CHOOSE THE CORRECT ANSWER :

1) An element $a \in G$ is called idempotent if $a^2 = ?$
   a. a^3
   b. a
   c. e
   d. Otherwise

2) If G be a group with identity e ,find x for the equation that $xab = c$
   a. $cb^{-1}a^{-1}$
   b. $c^{-1}b^{-1}a$
   c. cab
   d. cab^{-1}

3) If $a * e = a \; for \; all \; a \epsilon G,$ then e is called a
   a. Identity
   b. Left Identity
   c. Right Identity
   d. Group

4) If A is an permutation, $A = \{1,2,3,4\}$ for $f: A \rightarrow A$ given by $f(1) = 2,$ $f(3) = 4, f(2) = ?$
   a. 1
   b. 2
   c. 3
   d. 4

5) A cycle of length two is called a
   a. Disjoint
   b. Transposition
   c. Permutation
   d. Otherwise

6) Let us consider p be the permutation , Then p is a product of 4 transpositions Hence p is an
   a. Odd permutation
   b. Even permutation
   c. Both are true
   d. Both are false

7) Determine which of the following statements is true
   a. Every permutation is a cycle.
   b. Product of two cycles is a cycle
   c. Every cycle is a permutation
   d. Any finite group is abelian.

8) Find which one is not group
   a. $(N,+)$
   b. $(Z_n,\oplus)$
   c. $Z_n-\{0\}$
   d. $(Z,+)$

9) If G be a group is said to be abelian if $ab=?$
   a. a^2
   b. $-ba$
   c. ab^{-1}
   d. ba

10) If $f:R\to R$ is given by $f(x)=x+3$ and $g:R\to R$ is given by $g(x)=2x$, f and g are bijections then $f\circ g(x)=?$
   a. $3x+2$
   b. $2x+6$
   c. $2x+3$
   d. $x+3$

ANSWERS: 1) b 2) a 3) c 4) a 5) b 6) b 7) c 8) a 9) d 10) c

2 Marks :

11) Define group.
12) Define abelian group.
13) If G be a group , Let $a,b\in G$. Then $(ab)^{-1}=b^{-1}a^{-1}$ and $(a^{-1})^{-1`}=a$.
14) Show that in a group $x^2=x$ iff $x=e$.
15) Define left identity and right identity.
16) Define permutation.
17) Define symmetric group.
18) Define order of G
19) Define disjoint.
20) If $A=\{1,2,3,4,5\}$ consider the cycle of length 4 is given by $p=(2\ \ 4\ \ 5\ \ 1)$.

5 Marks :

21) Prove that The set of all n[th] roots of unity with usual multiplication is a group.
22) Prove that $(Z_n,\oplus)$ is a group.

23) Prove that Let G be a group. Then

(I) Identity element of G is unique.

(II) For any $a \in G$, the inverse of a is unique.

24) If G be a group and $a, b \in G$. Then the equations $ax = b$ and $ya = b$ have the unique solutions for x and y in G.

25) If G be a group in which $(ab)^m = a^m b^m$ for three consecutive integers and for all $a, b \in G$. Then G is abelian.

26) Let G be a non empty set with an associative binary operation defined on it such that there exists a left identity e in G and each element $a \in G$ has a left inverse a' with respect to e.Then$G$ is a group.

27) If G be a non empty set with an associative binary operation defined on it such that the equations $ax = b$ and $ya = b$ have the unique solutions for x and y in G. Then G is a group.

28) Prove that Any permutation can be expressed as a product of transpositions.

29) If A_n be the set of all even permutations in S_n, prove that A_n is a group containing $n!/2$ permutations.

30) If $G = \left\{\frac{z}{z} \in C \ and \ |z| = 1\right\}$. Prove that G is a group under usual multiplication.

10 Marks :

31) If $(a + ib)(c + id) = (ac - bd) + i(ad + bc)$,prove that C^* is a group under usual multiplication.

32) If G be the set of all real numbers except -1, Define $*$ on G by $a * b = a + b + ab$,then$(G,*)$ is a group.

33) Let n be a prime. Then $Z_n - \{0\}$ is a group under multiplication modulo n.

34) If $G = \{(a, b)/a \in R^*, \ b \in R\}$, Then G is a group under the operation $*$ defined by $(a, b) * (c, d) = (ac, bc + d)$.

35) Prove that

(I) $a^m a^n = a^{m+n} \ m, n \in Z$.

(II) $(a^m)^n = a^{mn}, m, n \in Z$.

36) If G be a finite set with an associative binary operation defined on G in which both cancellation laws hold good, Then$G$ is a group.

37) Prove that any permutation can be expressed as a product of disjoint cycles.

38) If a permutation $p \in S_n$ is a product of r transpositions and also a product of s transpositions then either r and s are both even or both odd.

39)Prove that the set of all positive integers less than n and prime to it is a group under multiplicationmodulo n.

40) Prove that

(I) The product of two odd permutations is an even permutations.

(II) The product of an even permutation and an odd permutation is an odd permutation.

(III) The inverse of an even permutation is an even permutation.

(IV) The inverse of an odd permutation is an odd permutation.

UNIT – II

CHOOSE THE CORRECT ANSWER :

1) If G be any group. Then $\{e\}$ and G are subgroups of G, They are called?
   a. Subgroup
   b. Symmetric group
   c. Improper subgroup
   d. normal subgroup

2) Find the order of -1 and 3 in $(R^*, .)$
   a. Infinite and 2
   b. 2 and infinite
   c. 3 and 1
   d. 1 and 3

3) Determine which one of the following is true?
   a. The order of e is zero.
   b. If in a group every element is of finite order then the group is finite.
   c. In a finite group the order of every element is finite.
   d. In an infinite group the order of every element is infinite.

4) Find the order of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$
   a. 3
   b. 4
   c. 2
   d. 1

5) Which one of the following statement is false?
   Let G be a group and H be a subgroup of G,
   a. Hitself is a left coset of H.
   b. The identity element belongs to every left cosets of H.
   c. $a \in H$ iff if $aH = H$.
   d. If $b \in Ha$ then $Ha = Hb$.

6) Find the number of elements in the cyclic subgroup $\langle 2 \rangle \; in \; (Z_{18}, \oplus)$

   a. 9
   b. 5
   c. 4
   d. 16

7) If A and B be cyclic groups of order m and n,respectively. Then $A \times B$ is cyclic iff :

   a. m divides n
   b. n divides m
   c. $m = n$
   d. m and n are relatively prime

8) Every cyclic group is an

   a. Infinite subgroup
   b. Abelian group
   c. Monoid
   d. Commutative semigroup

9) The number of generators of cyclic group of order 219 is

   a. 144
   b. 124
   c. 56
   d. 218

10) If a group G of order 20 is

   a. Unsolvable
   b. Solvable
   c. 1
   d. Not determined

ANSWERS: 1) c 2) b 3) c 4) b 5) b 6) a 7) d 8) b 9) a 10) b

2 Marks :

11) Define subgroup with give an example
12) Define cyclic group
13) Prove that any cyclic group is abelian
14) Define order of an element
15) Define left coset of a subgroup
16) If $a \in R^*$ and ,Let $H = \{a^n / n \in z\}$ then H is a subgroup of R^* .
17) Let H be a subgroup of G,Then the identity element of H is same as that of G.
18) Let G be a group and H be a subgroup of G. Then $a \in bH \Rightarrow a^{-1} \in Hb^{-1}$
19) Define index
20) State Fermat's theorem.

5 Marks :

21) A non empty subset H of a group G is a subgroup of G iff

$$a, b \in H \Rightarrow ab^{-1} \in H.$$

22) If H and K are subgroups of a group G, then is $H \cap K$ is also a subgroup of G.

23) Let H be a non empty finite subset of G. If H is closed under the operation in G then H is a subgroup of G

24) Prove thatAny subgroup of cyclic group is cyclic.

25) Let H be a subgroup of G, Then the number of left cosets of H is the same as the number of right cosets.

26) Let G be a group and a be an element of order $n$in G. Then $a^m = e$ iff $n$divides m.

27) If H and K be two subgroups of G of finite index in G, then $H \cap K$ is a subgroup of finite index.

28) If H and K be two finite subgroups of a group G, then $|HK| = \frac{|H||K|}{|H \cap K|}$.

29) Let G be a group prove that $= \{a/a \in G \text{ and } ax = xa \text{ for all } x \in G\}$.

30) State and prove Euler's theorem.

10 Marks :

31) The union of two subgroups of a group G is a subgroup iff one is contained in the other.

32) Let A and B be a two subgroups of a group G, then AB is a subgroup of G iff $AB = BA$.

33) Let G be a group and H be a subgroup of G, then

(i) $a \in H \Rightarrow aH = H$

(ii) $aH = bH \Rightarrow a^{-1}b \in H$

(iii) $a \in bH \Rightarrow a^{-1} \in Hb^{-1}$

(iv) $a \in bH \Rightarrow aH = bH$.

34) Let G be group and $a, b \in G$, then

(i) Order of a = Order of a^{-1}

(ii) Order of a = Order of $b^{-1}ab$

(iii) Order of ab = Order of .

35) State and prove Lagrange's theorem.

36) (i) every group of prime order is cyclic

(ii) a group G has no proper subgroups if it is a cyclic group of prime order.

37) a) If G be a group and $a \in G$, Then the order of a is the same as the order of the cyclic group generated by a.

b) Prove that a finite group every element is of finite order.

38) If G be a group and let a be an element of order $n$in G. Then the order of a^s, where $0 < s < n \text{ is } n/d$ where d is the g.c.d of $n$and s.

39) a) If G is a finite group with even number of elements then G contains at least one element of order 2.

b) Prove that the order of a permutation p is the $l.c.m$ of the lengths if its disjoint cycles.

40) Prove that the order of any element of a finite group G divides the order of G.

UNIT III

CHOOSE THE CORRECT ANSWER :

1) A subgroup H of G is called a normal subgroup of G if $aH =$? For all $a \in G$.
   a. ah
   b. Ha
   c. ha
   d. aH

2) Find the order of quotient group is $Z_6/\langle 3\rangle$
   a. 5
   b. 6
   c. 3
   d. 2

3) Which of the following statements are true?
   a. $Z/nZ$is cyclic.
   b. S_n/A_n is abelian
   c. Both are true
   d. None of these

4) If $f : G \to G'$ is an isomorphism, if G is cyclic then G' is also __________?
   a. Normal subgroup
   b. Generator group
   c. Cyclic
   d. Subgroup

5) Any finite group is isomorphic to a group of permutations is called an ?
   a. Cayley's theorem
   b. Lagranges theorem
   c. Fundamental theorem
   d. Fermat's theorem

6) If $(Z_4, \oplus)$, here 1 is a generator of this cyclic group. If $f(1) = 3$ then $f(2) =?$
   a. 1
   b. 3
   c. 2
   d. 4

7) If $f : G \to G'$ be a homomorphism, then if f is onto, it is called an ?
   a. Isomorphism
   b. Epimorphism
   c. Monomorphism
   d. Canonical homomorphism

8) If f is 1-1 is called an ?
   a. Isomorphism
   b. Epimorphism
   c. Monomorphism
   d. Canonical homomorphism

9) If f it is a homomorphism, find the kernel $f : (Z, +) \to (R^*, .)$is given by $f(x) = 3^*$
   a. $\ker f = 2Z$
   b. $\ker f = A_n$
   c. $\ker f = R^+$
   d. $\ker f = \{0\}$

10) Determine which of the following statement is true?
   a. $(Z, -)$ is a group
   b. $(Z_n, \oplus)$,is a group
   c. A subgroup of a non abelian group is non abelian
   d. $(Z_n, \odot)$,is a group

ANSWERS: 1) b 2) c 3) c 4) c 5) a 6) c 7) b 8) c 9) d 10) b

2 Marks :

11) Define normal subgroup.
12) Every subgroup of an abelian group is a normal subgroup
13) The centre H of a group G is a normal subgroup of G.
14) Determine the order of the quotient group is $Z/3Z$.
15) Define isomorphism.
16) If $f: G \to G'$ is an isomorphism. If G is abelian, then G' is also abelian.
17) Prove that $(Z_4, \oplus)$ is not isomorphic to V_4.
18) Define homomorphism.
19) Define kernel of f.
20) If $f: G \to G'$ be a homomorphism, then f is 1-1 iff $\ker \mathrm{f} = \{e\}$.

5 Marks :

21) If H be a subgroup of index 2 in a group G. Then H is a normal subgroup of G.
22) $M$and$N$ are normal subgroups of a group G such that $M \cap N = \{e\}$. Show that every element of M commutes with every element of N.
23) A subgroup N of G is normal iff the product of two right cosets of N is again a right cosetof N.
24) Let N be a normal subgroup of a group G. Then G/N is a group under the operation defined by $N_a N_b = N_{ab}$.
25) Let $f: G \to G'$ be an isomorphism. Then
    (i) $f(e) = e'$where$e$ and e' are the identity elements of G and G' respectively. (ie)in an isomorphism identity is mapped onto identity.
    (ii) $f(a^{-1}) = [f(a)]^{-1}$.
26) Let $f: G \to G'$ be an isomorphism. If G is cyclic then $G'$is also cyclic.
27) Let G be any group. Find $f: G \to G'$ given by $f(x) = x^{-1}$ is an isomorphism $\Leftrightarrow G$ is abelian.
28) For any group G,
    (i)$Aut\ G$is a group under composition of functions.
    (ii)$I(G)$ is a normal subgroup of $Aut\ G$.
29) Show that $Aut\ Z_8 \cong V_4$.
30) Let $f: G \to G'$ be a homomorphism.Then the kernel $K$of f is a normal subgroup of G.

10 Marks :

31) If N be a subgroup of G. Then the following are equivalent.
    (i) $N$is a normal subgroup of G.
    (ii) $aNa^{-1} = N\ for\ all\ a \in G$.
    (iii) $aNa^{-1} \subseteq N\ for\ all\ a \in G$.
    (iv) $ana^{-1} \in N\ for\ all\ n \in N\ and\ a \in G$.
32) Prove that Isomorphism is an equivalence relation among groups.
33) a) Any infinite cyclic group g is isomorphic to $(\mathrm{Z}, +)$.
    b) Any finite cyclic group of order n is isomorphic to $(Z_n, \oplus)$.
34) State and prove Cayley's theorem.
35) Let G be a cyclic group generated by a. Let $f: G \to G$ be a mapping such that $f(xy) = f(x)f(y)$. Then f is an automorphism of G iff $f(a)$ is a generator of G.
36) Let $f: G \to G'$ be a homomorphism. Then
    (I) If H is a subgroup of G then $f(H)$ is a subgroup of G'.
    (II) If H is normal in G, then $f(H)$ is normal in $f(G)$.
    (III) If H' is a subgroup of G', then $f^{-1}(H')$ is a subgroup of G.
    (IV) If H' is normal in $f(G)$ then $f^{-1}(H')$ is normal in G.
37) State and prove fundamental theorem of homomorphism.

38) a) show that the centre$H$ of a group G is a normal subgroup of G.
b) show that if H and N are subgroups of a group G and N is normal in G,then $H \cap N$ is normal in G.

39) If $f: G \to G'$ be an homomorphism, let $a \in G$, then the order of a is equal to the order of $f(a)$.(i.e) isomorphism preserves the order of each element in a group.

40) Show that if H is subgroup of G and N is a normal subgroup of G then HN is a subgroup of G.

UNIT IV

CHOOSE THE CORRECT ANSWER :

1) In a group of $(R, +)$,the unique additive inverse of a is denoted by ___?
   a. a
   b. 0
   c. $-a$
   d. a'

2) $\{0\}$with binary operations "+" and "." Defined as $0 + 0 = 0,\ 0.0 = 0$ is a ring,is called an ?
   a. Null ring
   b. Semiring
   c. Commutative ring
   d. Quotient ring

3) A ring is called an Boolean ring if $a^2 =$?
   a. $-a$
   b. a'
   c. a
   d. a''

4) An example of an infinite commutative ring without identity is
   a. $(Z, +, .)$
   b. $(Z_n,\ \oplus, \otimes)$
   c. $(2Z, +,\ .)$
   d. $M_2(R)$

5) Determine which of the following is true?
   a. In any integral domain every non zero element has an inverse
   b. $3Z$ is an integral domain
   c. Every ring has a multiplicative identity
   d. Any field is an integral domain

6) Determine which of the following is false?

a. Z is an ideal of R
b. R has no proper ideals
c. Q is a principal ideal domain
d. Z is a principal ideal domain

7) Any ordered integral domain is of characteristic
   a. 1
   b. 0
   c. Infinite
   d. Prime

8) The algebraic structure which is not a ring is
   a. $(Z,+,.)$
   b. $(Q,+,.)$
   c. $(Z_n, \otimes, \oplus)$
   d. $(Z_n, \oplus, \otimes)$

9) In $(Z,+.)$ …..
   a. 1 is the only unit
   b. -1 is the only unit
   c. 1 and -1 are the only units
   d. There is no unit

10) The kernel of the homomorphism $f:(R^*,.) \to (R^+,.)$ defined by $f(x) = |x|$ is
   a. $\{1\}$
   b. $\{-1\}$
   c. $\{0\}$
   d. $\{1,-1\}$

ANSWERS: 1) c 2) a 3) c 4) c 5) d 6) a 7) b 8) c 9) c 10) d

2 Marks :

11) Define ring.
12) Define ring of Gaussian integers.
13) The set R of all matrices of the form $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ where $a, b \in R$ is a ring under matrix addition and matrix multiplication.
14) If R be a ring and $a, b \in R$, then $a(-b) = (-a)b = -(ab)$.
15) Define isomorphism.
16) If $f : C \to C$ defined by $f(z) = \bar{z}$ is an isomorphism.
17) Define skew field.
18) Define integral domain.

19) Prove that Any unit R cannot be a zero divisor.

20) Define characteristic of the ring.

5 Marks :

21) If R is a ring such that $a^2 = a$ for all $a \in R$, prove that
   (i) $a + a = 0$
   (ii) $a + b = 0 \Rightarrow a = b$
   (iii) $ab = ba$
22) If R be a ring with identity ,the set of all units in R is a group under multiplication.
23) In a skew field R,
   (i) $ax = ay,\ a \neq 0 \Rightarrow x = y$
   (ii) $xa = ya,\ a \neq 0 \Rightarrow x = y$
   (iii) $ax = 0 \Leftrightarrow a = 0\ or\ x = 0$ (by using cancellation laws in ring)
24) Prove that a ring R has no zero divisors iff cancellation law is valid in R.
25) If R be a commutative ring with identity 1, then R is an integral domain iff the set of non zero elements in R is closed under multiplication.
26) Prove that Z_n is a field iff n is prime.
27) Prove that the set F of all real numbers of the form $a + b\sqrt{2}$ where $a, b \in Q$ is a field under the usual addition and multiplication of real numbers.
28) Prove that the only idempotent elements of an integral domain are 0 and 1.
29) If the additive group of a ring R is cyclic, prove that R is commutative .deduce that a ring with 7 elements is commutative.
30) A non empty subset S of a field F is a subfield iff
   (i) $a, b \in S \Rightarrow a - b \in S$ and
   (ii) $a, b \in S$ and $b \neq 0 \Rightarrow ab^{-1} \in S$

10 Marks :

31) Prove that $(Z_n,\ \oplus,\ \odot)$ is a ring.
32) a) Prove that Z_n is an integral domain iff n is prime.
   b) If Rbe a commutative ring with identity, then R is an integral domain iff cancellation laws valid in R.
33) a) Prove that any finite integral domain is a field.
   b) Prove that a finite commutative ring R without zero divisors is a field.
34) If R and R' be rings and $f : R \to R'$ be an isomorphism, then
   (i) R is a commutative $\Rightarrow R'$ is commutative
   (ii) $R$is ring with identity $\Rightarrow R'$ is a ring with identity.
   (iii) R is an integral domain $\Rightarrow R'$ is an integral domain
   (iv) $R$is a field $\Rightarrow R'$ is a field.
35) a) Prove that A field has no proper ideals.
   b) If R be a commutative ring with identity, then R is a field iff R has no proper ideals.

36) a) Let R be a ring with identity 1, if 1 is an element of finite order in the group $(R, +)$ then the order of 1 is the characteristic of R. If 1 is of infinite order, the characteristic of the ring is 0.

b) The characteristic of an integral domain D is either 0 or a prime number.

37) a) In an integral domain D of characteristic p, the order of every element in the additive group is p.

b) A non empty subset S of a ring R is a subring iff, $a, b \in S \Rightarrow a - b \in S$ and $ab \in S$

38) If R be a ring and I be a subgroup of $(R, +)$, the multiplication in R / I given by $(I + a)(I + b) = I + ab$ is well defined iff I is an ideal of R.

39) If A be any abelian group, let $Hom\ (A)$ be the set of all endomorphism of A. Let $f, g \in Hom\ (A)$. Define $f + g\ by\ (f + g)(x) = f(x) + g(x)$ and $fg = f_\circ g$. Then $Hom\ (A)$ is a ring.

40) a) Prove that in the case of a ring with identity the axiom $a + b = b + a$ is a redundant.(ie) the axiom $a + b = b + a$ can be derived from the other axioms of the ring.

b) Prove that the only isomorphism $f : Q \to Q$ is the identity map.

UNIT V

CHOOSE THE CORRECT ANSWER :

1) A homomorphism of a ring onto itself is called an _________?
   a. Epimorphism
   b. Monomorphism
   c. Isomorphism
   d. Endomorphism

2) If an ideal $M \neq R$ is said to be maximal ideal of R if whenever U is an ideal of R such that $M \subseteq U \subseteq R$ either $U = M\ or\ U =$?
   a. $U = R$
   b. $M = R$
   c. $U \neq R$
   d. Otherwise

3) In an equation of homomorphism to find the kernel of $f : C \to C$ defined by $f(z) = \bar{z}$
   a. $\ker f = \{0\}$
   b. $\ker f = nz$
   c. $\ker f = z$
   d. Otherwise

4) Which of the following statement is true?

a. A homomorphic image of an integral domain is an integral domain
b. Every isomorphism is a homomorphism
c. A homomorphic image of a skewfield is a skewfield
d. Homomorphic image of a field is a field

5) Which of the following statement is false?

a. R is a field of quotients of R
b. Q is a field of quotients of Z
c. R is a field of quotients of Z
d. If D is any field then the field of quotients of D is isomorphic to D

6) Two elements a and b of an Euclidean domain R is said to be relatively prime if their g.c.d is

a. 1
b. 0
c. 2
d. Otherwise

7) Find the g.c.d of ring Gaussian integers of $11 + 7i$

a. $7 - i$
b. $3 + 4i$
c. i
d. 0

8) If $f : R \to R'$ is defined by $f(a) = 0 \; for \; all \; a \in R$ is a homomorphism then f is called the

a. Trivial homomorphism
b. Non trivial homomorphism
c. Natural homomorphism
d. None of these

9) If $f : R \to R'$ is a homomorphism,and f is 1-1 then f is called a

a. Epimorphism
b. Monomorphism
c. Endomorphism
d. None of these

10) Find the g.c.d of ring Gaussian integers of $1 + 6i$

a. i
b. $3 + 4i$
c. $7 - i$
d. $4 - 3i$

ANSWERS: 1) d 2) a 3) a 4) b 5) c 6) a 7) c 8) a 9) b 10) c

2 Marks:

11) Define maximal ideal
12) Define prime ideal
13) (3) is a prime ideal of Z.
14) Define homomorphism.
15) Let R and R' be rings and $f: R \to R'$ be a homomorphism, Then $f(0) = 0'$.
16) The homomorphic image of an integral domain need not be an integral domain.
17) Define ordered integral domain.
18) The field of complex numbers is not an ordered field.
19) Define unique factorization domain.
20) If R be an Euclidean domain,let $a, b \in R$ then $a \mid bc$and $(a, b) = 1 = a \mid c$.

5 Marks :

21) Let $f: R \to R'$ be a homomorphism. Let K be the kernel of f. Then K is an ideal of R.
22) The field of quotients F of an integral domain D is the smallest field containing D.(ie) If the F' is any other field containing D then F' contains a subfield isomorphic to F.
23) Let R be an integral domain. Let a and b be two non zero elements of R. Then a and b are associates iff $a = bu$ where u is a unit in R.
24) The ring of Gaussian integers $R = \{a + bi/\ a, b \in Z\}$ is an Euclidean domain,where $d(a + bi) = a^2 + b^2$.
25) Let R be an Euclidean domain and I be an ideal of R. Then there exists an element $a \in I$ such that $I = aR$. (ie) every ideal of an Euclidean domain is a principal ideal.
26) Any Euclidean domain R has an identity element.
27) Show that $1 + i$ is a prime element in the ring R of Gaussian integers.
28) Let a be a non zero element of an Euclidean domain R. Then a is a unit in R iff $d(a) = d(1)$.
29) Let R be an Euclidean domain. Then any two elements $a, b \in R$ have a g.c.d and it is of the form $ax + by$ where $x, y \in R$.
30) Let p be a prime element in an Euclidean domain R. Let $a, b \in R$.
    Then $p \mid ab \Rightarrow p \mid a\ or\ p \mid b$.

10 Marks :

31) Let R be a commutative ring with identity. An ideal M of R is maximal iff R/M is a field.
32) If R be any commutative ring with identity. Let P be an ideal of R.Then$P$ is a prime ideal $\Leftrightarrow R/P$is anintegral domain.
33) IfR be a commutative ring with identity. Then every maximal ideal of R is a prime ideal of R.
34) State and prove fundamental theorem of homomorphism.

35)a)If R be a Euclidean domain, let a and b be two non zero elements of R.Then

i) $b \text{ is not a unit in } R \Rightarrow d(a) < d(ab)$

ii) $b \text{ is a unit in } R \Rightarrow d(a) = d(ab)$.

b) Prove that any Euclidean domain R is a principal ideal domain.

36) Prove that any Euclidean domain R is a U.F.D
37) If D be any order integral domain then the relation defined the following properties
    i) $<$ is transitive (i.e) $a < b$ and $b < c \Rightarrow a < c$.
    ii) $b < c \Rightarrow a + b < a + c$
    iii) $b < c, a > 0 \Rightarrow ab < ac$.
38) If $f : R \rightarrow R'$ be rings of homomorphism ,then prove that
    i) If S is an ideal of R ,then $f(S)$ is an ideal of $f(R)$.
    ii) If S' is a subring of R', then $f^{-1}(S')$ is a subring of R.
    iii) If S' is an ideal of $f(R)$, then $f^{-1}(S')$ is an ideal of R.
39) Prove that if $f : R \rightarrow R'$ be a homomorphism, let K be the kernel of f. Then $K$is an ideal of R.
40) a) Prove that any ordered integral domain the square of any non zero element is positive.

    b) Prove that any ordered integral domain D is of characteristic 0.