ஸ்ரீ-ல-ஸ்ரீ காசிவாசி சுவாமிநாத சுவாமிகள் கலைக் கல்லூரி

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S.K.S.S ARTS COLLEGE, THIRUPPANANDAL - 612504

QUESTION BANK

Title of the Paper

DIFFERENTIAL CALCULUS AND TRIGONOMETRY

COURSE – I B.Sc., Maths

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DIFFERRENTIAL CALCULUS AND TRIGONOMETRY

CORE COURSE I

Objectives:

1. To inculcate the basics of differentiation and their applications.
2. To introduce the notion of curvatures, Evolutes & Involutes and polar co-ordinates.
3. To understand the basic concepts of Trigonometry

UNIT I

Methods of Successive Differentiation – Leibnitz,s Theorem and its applications- Increasing & Decreasing functions –Maxima and Minima of function of two variables.

UNIT II

Curvature – Radius of curvature in Cartesian and in Polar Coordinates – Centre of curvature–Evolutes & Involutes

UNIT III

Expansions of sin (nx), cos (nx), tan (nx) – Expansions of sin nx, cos nx – Expansions of sin(x), cos(x), tan(x) in powers of x.

UNIT IV

Hyperbolic functions – Relation between hyperbolic & Circular functions- Inverse hyperbolic functions.

UNIT V

Logarithm of a complex number –Summation of Trigonometric series – Difference method- Angles in arithmetic progression method –Gregory's series

TEXT BOOKS:

1. S.Narayanan and T.K.Manicavachagom Pillai, **Calculus Volume I**, S.Viswanathan (Printers&Publishers) Pvt Limited , Chennai -2011.

2. S.Arumugam & others, **Trigonometry and Fourier series**, New Gamma Publications -1999

REFERENCE(S)

1. S.Arumugam and Isaac, Calculus, Volume1, New Gamma Publishing House, 1991.

2. S. Narayanan, T.K. Manichavasagam Pillai, Trigonometry, S. Viswanathan Pvt Limited, and Vijay Nicole Imprints Pvt Ltd, 2004.

UNIT – I

CHOOSE THE CORRECT ANSWEERS

1. The n^{th} derivative of e^{ax} is

A) $a^n e^{ax}$

B) 0

C) $a^{n+1} e^{ax}$

D) $a^{-n} e^{ax}$

2. If the values of f(X) increases as x increase then it's known as

A) Decreasing function

B) Increasing function

C) Derivative function

D) None of these

3. If f(x) as a minimum if

A) $f''(a) < 0$

B) $f''(a) = 0$

C) $f''(a) \leq 0$

D) $f''(a) > 0$

4. If f has a maximum and the corresponding value of f is?

A) Maximum value

B) Minimum value

C) None

D) A & B

5. If y=x then the value of $\frac{d^2y}{dx^2}$

A) 2

B) 8

C) 0

D) 1

6. The value of $1 - \cos\theta$ is

A) $\sin^2(\theta/2)$

B) $\cos^2(\theta/2)$

C) $2\sin^2(\theta/2)$

D) None of these

7. If $\boldsymbol{u} = \sin 3x$ then the value of u_n is

A) $3^{n+1}\left\{\sin\left(3x + \frac{(n-1)\pi}{2}\right)\right\}$

B) $3^n\left\{\sin\left(3x + \frac{n\pi}{2}\right)\right\}$

C)$\sin(3x + n/2)$

D) A & B

8. $u = x^2 e^{ax}$ The value of v_2 is

A) 2x

B) x^2

C) 2

D) 0

9. If x^3+3x^2+3x+7 then the following function is?

A) Increasing function

B) Decreasing function

C) A & B

D) None of these

10. If f(x) =3x+1 then the value of $f'(x)$ is

A) 1

B) 2

C) 3

D) 0

ANSWERS:

1. A 2.B 3.D 4.A 5.C 6.C 7.B 8.C 9.A 10.C

TWO MARK QUESTIONS

11. Prove that $f(x) = x^2$ is an increasing function for $x \geq 0$.

12. If $y = e^{5x}$ find y_n?

13. If $y = e^{2x}$ find y_{10}?

14. The n^{th} derivative of $\log(ax + b)$.

15. Show that $\frac{d^n}{dx^n}[\cos(ax + b)] = a^n \cos\left[\frac{n\pi}{2} + ax + b\right]$.

16. Find the n^{th} derivative of $\log\left(\frac{x+a}{x-a}\right)$.

17. Define maxima and minima.

18. Write down the formula Leibnitz theorem?

19. Define decreasing and increasing function.

20. To find n^{th} derivative of $\cos^3 2x$.

FIVE MARK QUESTIONS

21. $\frac{d^n}{dx^n}[e^{ax}\sin(bx+c)] = e^{ax}r^n\sin(n\theta+bx+c)$ Where $r=\sqrt{a^2+b^2}$ and$\theta=\tan^{-1}\left(b/a\right)$.

22. Find$D^n\left(\frac{1}{(x+1)(x+3)}\right)$.

23. Find n^{th} derivatives of$\frac{x-2}{(x+2)(x-1)^2}$.

24. Find n^{th} derivatives of $\sin^2 x$

25. Find the maximum and minimum of function f(x) =$2x^3-3x^2-36x+10$.

26. Find the maximum and minimum of function f(x) = $x^2+y^2+6x+12$.

27. Check whether the following function $\sin x$ is decreasing on$(0, \pi/2)$.

28. for what values of x is $2x^3-15x^2-84x+7$ a decreasing function?

29. Find the range of values of x for which the function $x^3-6x^2-36x+7$ is increasing with x.

30. Prove that polynomial $2x^3+3x^2-12x+7$ is positive if x>1.

TEN MARK QUESTIONS

31. If $x=a(t-\sin t)$ & $y=a(1+\cos t)$ find$\frac{d^2y}{dx^2}$?

32. If $y=e^{\tan^{-1}x}$ show that $(1+x^2)y_2+(2x-1)y_1=0$.

33. Find n^{th} derivative of $\frac{3x-1}{x(x+1)(x-1)}$.

34. Find n^{th} derivative of $\frac{x^2-4}{(x+1)(x+4)}$.

35. Find $D^n\left[\frac{x+5}{x^2-2x-3}\right]$

36. Find n^{th} derivative of $\sin^4 x\cos^3 x$.

37. Find n^{th} derivative of $\sin^3 x\cos^5 x$.

38. Find n^{th} derivative of $x^2e^{3x}\sin 4x$ using Leibnitz theorem?

39. If$y=x^2e^x$ then show that $y_n=\frac{1}{2}n(n-1)y_2-n(n-2)y_1+\frac{1}{2}(n-1)(n-2)y$

40. Discuss the maximum and minimum of the function $x^3y^2(6-x-y)$ satisfying the condition x>0, y>

UNIT – II

CHOOSE THE CORRECT ANSWER

1. The name of the Evolutes of an ellipse is ____?

A) Centroid

B) Astroid

C) Asteroid

D) Cycloid

2. The curvature of a curve is equal to _____?

A) Reciprocal of radius of curvature

B) Radius of curvature

C) Twice the radius of curvature

D) None of these

3. Involutes are also known as _____?

A) Evolute

B) Evoluent

C) Envelope

D) Tangent

4. What is the curvature of the curve $x^2+y^2=25$

A) 5

B) 25

C) 0.5

D) 0.2

5. What is the evolute of parabola called?

A) Cycloid

B) Spiral

C) Congruent parabola

D) Semi cubical parabola

6. Definition of evolute of a curve is _____?

A) Locus of centre of the given curve

B) Locus of centre of tangential curve

C) Locus of circumferential point on the curve

D) Locus of tangent to the curve

7. What is the curvature of straight line?

A) Infinity

B) One

C) Zero

D) Length of the straight line

8. Number of possible evolutes for a curve is _____?

A) Two

B) Equal to radius

C) One

D) Infinity

9. The curvature of a plane curve at K is _____?

A) One

B) $\frac{d\varphi}{ds}$

C) Zero

D) Infinity

10. What is the radius of curvature of the curve $xy=c^2$ at (c,c)?

A) c

B) 2

C) 2c

D) $\sqrt{2}\,c$

ANSWERS:

1.B 2. A 3.B 4.D 5.D 6.A 7.C 8.C 9.B 10.D

TWO MARK QUESTIONS

11. Write down the formula for radius of curvature?

12. Define radius of curvature.

13. Define curvature.

14. If $y^2=4x$ find y_2.

15. Find the coordinates of the centre of curvature of the curve $y=x^2$ at the point (1/2, 1/4).

16. Write down the formula for centre of curvature?

17. If $y=3x^3+2x^2-3$ at (0, -3) to find $\bar{x}$.

18. To find $(\frac{dy}{dx})$ at (0, c) if $y = c\cos {}^{hx}/_{c}$.

19. What are evolutes and involutes?

20. How do you find the evolutes of a curve?

FIVE MARK QUESTIONS

21. Show that the radius of curvature of the curve $r^n = a^n \cos n\theta$ is $\frac{a^n r^{-n+1}}{n+1}$

22. Find the radius of curvature for the curve $x^3+y^3=3axy$ for the point of x=y=3a/2.

23. Find the radius of curvature at (x, y) for the curve $a^2y=x^3-a^3$.

24. Find the centre of curvature of the curve $y = 3x^3 + 2x^2 - 3$ at (0, -3).

25. Prove that the radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta)$; $y = a(1 - \cos\theta)$ is $4a\cos {}^{\theta}/_{2}$.

26. Explain centre of curvature?

27. Explain radius of the curvature?

28. Find the radius of curvature for the curve $y^2=x^3+8$ at (-2, 0)

29. Find ρ for the curve y=ccoshx/c at (0, c)

30. Prove that at the point $x = \pi/2$ of the curve $y = 4\sin x - \sin 2x$ is $\rho = \frac{5\sqrt{5}}{4}$.

TEN MARK QUESTIONS

31. Find the centre of curvature on the parabola $y^2=4ax$ at any point (x, y)

32. Find the equation of the evolutes of the parabola $y^2=4ax$.

33. Find the equation of the evolutes of the curve $x^{2/3} + y^{2/3} = a^{2/3}$

34. Find the equation of the evolutes of the ellipse $x^2/a^2 + y^2/b^2 = 1$

35. Find the radius of curvature for the curve $\sqrt{x} + \sqrt{y} = 1$ at (1/4, 1/4).

36. Find the radius of curvature of the curve $xy^2=a^3-x^3$ at (a, 0)

37. For the ellipse $x^2/a^2 + y^2/b^2 = 1$ show that the radius of curvature at an end of the major axis is equal to the semi-loctus rectum.

38. Show that the radius of curvature at any point on the curve $r = a(1 - \cos\theta)$ varies as $\sqrt{r}$

39. Find ρ for the curve$r = ae^{\theta \cot \alpha}$.

40. Show that the radius of curvature at any point on the curve $x = ae^{\theta}(\sin\theta - \cos\theta)$; $y = ae^{\theta}(\sin\theta + \cos\theta)$

UNIT – III

CHOOSE THE CORRECT ANSWEERS

1. Demoivre's theorem of $(\cos\theta + i\sin\theta)^n$ is

A) $\cos n\theta + i\sin n\theta$
B) $\cos n\theta - i\sin n\theta$
C) $-\cos\theta - i\sin\theta$
D) None of these

2. There are only _____ numbers of terms in the expansion of$\cos n\theta$ & $\sin n\theta$.

A) Infinite

B) Random

C) Finite

D) A & B

3. The value of $\cos^2\theta$ is

A) $\frac{1}{2}(1-\cos 2\theta)$

B) $\frac{1}{2}(1+\cos 2\theta)$

C) $1-\cos 2\theta$

D) None of these

4. The value of $\sin^2\theta$ is

A) $1+\cos 2\theta$

B) $\frac{1}{2}(1+\cos 2\theta)$

C) $\frac{1}{2}(1-\cos 2\theta)$

D) None of these

5. The value of $\cos^3\theta$ is

A) $\frac{1}{4}(\cos 3\theta - 3\cos\theta)$

B) $\frac{1}{3}(\cos 3\theta + 3\cos\theta)$

C) $\frac{1}{4}(\cos 3\theta + 3\cos\theta)$

D) None of these

6. The value of $\sin^3\theta$ is

A) $\frac{1}{4}(3\sin\theta - \sin 3\theta)$

B) $\sin\theta - \cos^4\theta$

C) None of these

D) $3\sin\theta - \sin 3\theta$

7. If $\frac{\sin x}{x} = \frac{863}{864}$ the value of x is

A) $2^0 46' 29''$

B) $3^0 46' 29''$

C) $4^0 46' 29''$

D) None of these

8. If $\frac{\tan\theta}{\theta} = \frac{2524}{2523}$ the value of x is

A) $1^0 56'$

B) $1^0 58'$

C) None of these

D) $1^0 57'$

9. The expansion of $\sin\theta$ & $\cos\theta$ are____?

A) Finite

B) Infinite

C) A & B

D) None of these

10. The expansions are valid only if θ is _____?

A) Ascending series

B) Descending series

C) Radians

D) A & C

ANSWERS:

1. A 2. C 3. B 4. C 5. C 6. A 7. C 8. B 9. B 10. C

TWO MARK QUESTIONS

11. Write down the formula for ascending powers of θ.

12. Write down the expansion of $\tan 5\theta$

13. Write down the expansion of $\sin n\theta$.

14. Write down the expansion of $\cos n\theta$.

15. Write down the expansion of $\tan n\theta$.

16. Expand $\tan 7\theta$ in terms of $\tan \theta$.

17. Evaluate $\lim_{x\to 0} \frac{\tan 2x - 2\tan x}{x^3}$

18. Define Demoivre's theorem.

19. Define Binomial theorem.

20. Expand $\tan 6\theta$ in powers of$\tan \theta$.

FIVE MARK QUESTIONS

21. Express $\frac{\sin 6\theta}{\sin \theta}$ in terms of$\cos \theta$.

22. Expand $\sin^7\theta$ in a series of sine of multiplies ofθ.

23. Expand $\cos 6\theta$ in terms of $\sin \theta$.

24. Show that $\cos 4\theta = 8\sin^4\theta - 8\sin^2\theta + 1$.

25. Prove that$\frac{\sin 7\theta}{\sin \theta} = 7 - 56\sin^2\theta + 112\sin^4\theta - 64\sin^6\theta$.

26. Prove that -$64\sin^7\theta = \sin 7\theta - 7\sin 5\theta + 21\sin 3\theta - 35\sin \theta$.

27. If $\frac{\sin x}{x} = \frac{863}{864}$ find an approximate value of x?

28. $\frac{\tan \theta}{\theta} = \frac{2524}{2523}$ Find θ approximately?

29. If $\cos\left(\frac{\pi}{3} + \theta\right) = 0.49$ show that $\theta = 39'5''$ nearly.

30. Evaluate$\lim_{x\to 0} \frac{\sin 2x - 2\sin x}{x^3}$.

TEN MARK QUESTIONS

31. Prove that $\cos^5\theta\sin^4\theta = \frac{1}{2^8}[\cos 9\theta + \cos 7\theta - 4\cos 5\theta - 4\cos 3\theta + 6\cos \theta]$.

32. Prove that $\sin^8\theta = \frac{1}{2^7}[\cos 8\theta - \cos 6\theta + 28\cos 4\theta - 56\cos 2\theta + 35]$.

33. Prove that $\sin^5\theta\cos^2\theta = \frac{1}{2^6}[\sin 7\theta - 3\sin 5\theta + \sin 3\theta + 5\sin \theta]$.

34. Determine a, b, c such that $\lim_{\theta\to 0} \frac{\theta(a + b\cos \theta) - c\sin \theta}{\theta^5} = 1$.

35. Prove that the length of a small circular arc is $\frac{1}{3}(8b - a)$.Where a is a length of the chord of the arc and b is that the chord of half the arc.

36. If $\sin\theta = 0.5033$ show that θ is approximately is approximately $30^0 13' 6''$.

37. Evaluate $\lim_{x\to 0} \frac{\tan x - \sin x}{\sin^3 x}$.

38. Prove that $32\sin^4\theta\cos^2\theta = \cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2$.

39. Show that $\cos^8\theta = \frac{1}{128}[\cos 8\theta + 8\cos 6\theta + 28\cos 4\theta + 56\cos 2\theta + 35]$.

40. Prove that $\frac{\sin 7\theta}{\sin\theta} = 64\cos^6\theta - 80\cos^4\theta + 24\cos^2\theta - 1$.

UNIT – IV

CHOOSE THE CORRECT ANSWERS

1. The value of $e^{i\theta}$ is

A) $\cos\theta + i\sin\theta$

B) $\cos\theta + \sin\theta$

C) $\cos\theta \sin\theta$

D) None of these

2. The value of $e^{-i\theta}$ is

A) $\tan hz$

B) $\cos\theta - i\sin\theta$

C) $\cot^{-1} hy$

D) $\text{cosec}^{-1}\theta$

3. The value of $\cos(ix)$ is

A) coshx

B) sinhx

C) tanhx

D) None of these

4. The value of $\sin(ix)$ is

A) icoshx

B) itanhx

C) isinhx

D) A & C

5. The value of $\tan(ix)$ is

A) sinhy

B) itanhx

C) A & B

D) coshy

6. The value of e^x is

A) $1+\frac{x}{2!}+\frac{x^2}{4!}+\frac{x^3}{6!}+\cdots$

B) $1-\frac{x}{1!}+\frac{x^2}{2!}-\cdots$

C) $1-\frac{x}{1!}-\frac{x^2}{2!}-\frac{x^3}{3!}-\cdots$

D) $1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$

7. The value of sinx is _____?

A) $x+\frac{2x}{1!}+\frac{3x}{2!}+\frac{4x}{3!}+\cdots$

B) $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots$

C) None of these

D) A & B

8. The value of cosx is_____?

A) $1-\frac{x^2}{2!}+\frac{x^4}{4!}+\cdots$

B) $1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$

C) A & B only

D) None of these

9. e^z Is a periodic function of period is _____

A) 2πi

B) 5πi

C) πi

D) 2π

10. The value of $\cosh^2 x + sinh^2 x$ is

A) cosh2x

B) sinh2x

C) tanh2x

D) sinhx

ANSWERS:

1. A 2.B 3.A 4.C 5.B 6.D 7.B 8.C 9.A 10.A

TWO MARK QUESTIONS

11. Define Euler's formula.

12. Define periodicity of exponential function.

13. Find $\frac{e^z - e^{-z}}{2}$.

14. Find $\frac{e^z + e^{-z}}{2}$.

15. Find sinhz.

16. Find cosh.

17. Find sinh2x.

18. Find $\cosh^2 x - sinh^2 x$.

19. Find $sech^2x$.

20. Find $cosech^2x$.

FIVE MARK QUESTIONS

21. Prove that $\sinh^{-1} x = \log_e\left(x + \sqrt{x^2 + 1}\right)$.

22. Prove that $\cosh^{-1} x = \log_e\left(x + \sqrt{x^2 - 1}\right)$.

23. Prove that $\tanh^{-1} x = \frac{1}{2}\log_e\left(\frac{1+x}{1-x}\right)$.

24. Prove that $\cosh 2x = \frac{1+\tanh^2 x}{1-\tanh^2 x}$

25. Prove that $\tanh 3x = \frac{3\tanh x + \tanh^3 x}{1+3\tanh^2 x}$

26. Prove that $\frac{1+\tanh x}{1-\tanh x} = \cosh 2x + \sinh 2x$

27. If $\tan\frac{x}{2} = \tan\frac{x}{2}$ show that cosxcoshx=1

28. Separate into real and imaginary parts of $\tan(x + iy)$

29. Separate into real and imaginary parts of $\tan^{-1}(x + iy)$

30. Show that $\cosh^5\theta = \cosh 5\theta + 5\cosh 3\theta + 10\cosh\theta$.

TEN MARK QUESTIONS

31. If $\tan\theta = \tanh x \coth y$ and $\tan\emptyset = \tanh x \tan y$ and show that and show that $\frac{\sin 2\theta}{\sin 2\emptyset} = \frac{\cosh 2x + \cos 2y}{\cosh 2x - \cos 2y}$.

32. If $\tan x/2 = \tanh y/2$ prove that $\sin hy = \tan x$ and $y = \log\tan(\pi/4 + x/2)$.

33. If $\sin(\theta + i\emptyset) = \tan(x + iy)$ show that $\frac{\tan\theta}{\tanh\emptyset} = \frac{\sin 2x}{\sinh 2y}$.

34. If $\cos(\theta + i\emptyset) = k(\cos\alpha + i\sin\alpha)$ prove that $\cos 2\theta + \cosh 2\emptyset = 2k^2$.

35. If $\sin(A + iB) = x + iy$ prove that

i) $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$

ii) $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 B} = 1$

36. Ifu $= \log\tan(\pi/4 + \theta/2)$ show that i) $\tanh\frac{u}{2} = \tan\frac{\theta}{2}$ ii) $\theta = -i\log\tan\left(\frac{\pi}{4} + i\frac{u}{2}\right)$

37. If $\cos(x + iy) = r(\cos\alpha + i\sin\alpha)$ prove that $y = \frac{1}{2}\log\left[\frac{\sin(x-\alpha)}{\sin(x+\alpha)}\right]$

38. If $\sin(\theta + i\emptyset)\sin(\alpha + i\beta) = 1$ show that

i) $\tanh^2\emptyset\cosh^2\beta = \cos^2\alpha$

ii) $\cosh^2\emptyset\tanh^2\beta = \cos^2\theta$

39. If $x = 2\cos\alpha\cosh\beta$ and $y = 2\sin\alpha\sinh\beta$ show that

i) $\sec(\alpha + i\beta) + \sec(\alpha - i\beta) = \frac{4x}{x^2+y^2}$ ii) $\sec(\alpha + i\beta) - \sec(\alpha - i\beta) = \frac{4iy}{x^2+y^2}$

40. If $\tan(\alpha + i\beta) = i, \alpha$ and β being real, prove that α is indeterminate and β is infinite.

UNIT – V

CHOOSE THE CORRECT ANSWER

1. The real part of $\log(x + iy)$ is _____?

A) $\frac{1}{2}\log(x^2 + y^2)$

B) $\frac{1}{2}\log(x^2 - y^2)$

C) $\frac{1}{2}\log(x^2y^2)$

D) $\log(x^2 + y^2)$

2. The imaginary part of $\log(x + iy)$ _____?

A) $\tan^{-1}x$

B) $i\tan^{-1}\left(\frac{y}{x}\right)$

C) $i\tan^{-1}\left(\frac{x}{y}\right)$

D) None of these

3. The value of $\log(-x)$ is

A) $\log x$

B) $\log i\pi$

C) $\log x + i\pi$

D) B & C

4. The value of $\log(1 + z)$ is

A) $z + \frac{z}{2} + \frac{z}{3} + \cdots$

B) 0

C) $2\pi i$

D) $z - \frac{z^2}{2} + \frac{z^3}{3} - \cdots$

5. The real value of log(i-3) is

A) $\frac{1}{2}\log(1^2 + 3^2)$

B) $\frac{1}{2}\log(1^2 + 3^2) = \frac{1}{2}\log(10)$

C) A & B

D) None of these

6. The real value of log(1+i) is

A) $\frac{1}{2}\log(1^2 + 1^2) = \frac{1}{2}\log(2)$

B) $\frac{1}{2}\log(2^2 + 4^2) = \frac{1}{2}\log(50)$

C) $\frac{1}{2}\log(x) = \frac{1}{2}\log(2x + 9)$

D) A & B

7. The real value of $\log(-1 + i)$ is

A) $\tan^{-1}(hx + c)$

B) $\cos\theta + i\sin\theta$

C) $\frac{1}{2}\log(1^2 + 1^2) = \frac{1}{2}\log 2$

D) A & B

8. The imaginary value of $\log(i - 3)$ is

A) $\tan^{-1} 2i$

B) $\sec hx + c$

C) $\tan^{-1}(ihx) + 2\pi$

D) $\tan^{-1}(-3) + 2n\pi$

9. The imaginary value of $\log(1 + i)$

A) $\tan^{-1}(1) + 2n\pi$

B) $\sec^{-1}(1) + 2\pi i$

C) A & B

D) None of these

10. The imaginary value of $\log(-1 + i)$

A) $\tan^{-1}(ax + c) + 2n\pi$

B) $\cos^{-1} ix + c$

C) $\tan^{-1}(-1) + 2n\pi$

D) None of these

ANSWERS:

1.A 2.B 3.C 4.D 5.B 6.A 7.C 8.D 9.A 10.C

TWO MARK QUESTIONS

11. Separate the real and imaginary parts of $\log(4 + 3i)$?

12. Find log (1+i).

13. Find the sum to n terms of the series $\sin\alpha + \sin 3\alpha + \sin 5\alpha + \cdots$

14. Define logarithm of a complex number?

15. Find $\log i$?

16. Find $\log 3$?

17. Write down the formula for the sum of sines of n angles in A.P?

18. Write down the formula for Gregory's series?

19. What is the value of $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$?

20. If n is an integer, what is the value of $\cos\left(\frac{n\pi}{2}\right)$.

FIVE MARK QUESTIONS

21. Explain principal and general value of the logarithm of a complex number?

22. Prove that the logarithm of a complex number is a many – valued function?

23. Find the real and imaginary parts of $\log(x + iy)$.

24. Show that $\log\left(\frac{a+ib}{a-ib}\right) = 2i\tan^{-1}\left(\frac{b}{a}\right)$.

25. Show that $\log_i i = \frac{4n+1}{4m+1}$ where m & n integers.

26. If |x|<$\sqrt{2}$-1 prove that $2\left(x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots\right) = \left(\frac{2x}{1-x^2}\right) - \frac{1}{3}\left(\frac{2x}{1-x^2}\right)^3 + \cdots$

27. Separate into real and imaginary parts of $\log(1 + i)^{1-i}$.

28. Find the sum to n terms of the series $\cos\alpha + \cos 5\alpha + \cos 9\alpha + \cos 13\alpha + \cdots$

29. Sum the series up to n terms $\tan\alpha + 2\tan 2\alpha + 2^2\tan 2^2\alpha + \cdots + 2^{n-1}\tan 2^{n-1}\alpha$.

30. Prove that $i^i = e^{-(4n+1)\pi/2}$ where n is an integer?

TEN MARK QUESTIONS

31. Find $S_n = sinhx + \sinh(x + y) + \sinh(x + 2y) + \cdots + n$ terms.

32. Prove that real and imaginary parts of the principal value of $i^{\log(1+i)}$ is $e^{-\pi^2/8}\cos\left(\frac{\pi}{4}log2\right)$.

33. Find the value of$\log\left(\frac{1+\cos\alpha+i\sin\alpha}{\cos\alpha-1+i\sin\alpha}\right)$.

34. Find$\log\left\{\frac{\sin\{x-iy\}}{\sin(x+iy)}\right\}$.

35. To derive a formula for the sum of sines of n angles in A.P?

36. Find the sum to n terms of the series $\sin^3\alpha + \sin^3 2\alpha + \sin^3 3\alpha + \cdots$

37. Sum to n terms of the series $\cosh x + \cosh(x + y) + \cosh(x + 2y) + \cdots$

38. Show that$\cot\alpha\sin\theta - \frac{1}{3}\cot^3\alpha\sin 3\theta + \frac{1}{5}\cot^5\alpha\sin 5\theta - \cdots = \frac{1}{2}\tanh^{-1}(\sin\theta\sin 2\alpha)$.

39. If$C = \cos^2\theta - \frac{1}{3}cos^3\theta cos3\theta + \frac{1}{5}cos^5\theta cos5\theta - \cdots\infty$show that $\tan 2c = 2\cot^2\theta$.

40. Show that $\cos\theta - \frac{1}{3}\cos 3\theta + \frac{1}{5}\cos 5\theta - \cdots = \frac{\pi}{4}$.