ஸ்ரீ-ல-ஸ்ரீ காசிவாசி சுவாமிநாத சுவாமிகள் கலைக் கல்லூரி

திருப்பனந்தாள் – 612504

S.K.S.S ARTS COLLEGE, THIRUPPANANDAL - 612504

QUESTION BANK

Title of the Paper

LINEAR ALGEBRA

COURSE – II B.Sc., Maths

Prepared by

S.BAKIYALAKSHMI M.Sc.,M.Phil., B.Ed.,

Assistant Professor

Department of Mathematics

CORE COURSE VIII

LINEAR ALGEBRA

Objectives

1. To facilitate a better understanding of vector space

2. To solve problems in linear algebra

UNIT – I   Vector spaces:

Vector spaces – definition and examples – subspaces – linear transformation – span of a set.

UNIT – II   Basis and dimension:

Linear independence – Basis and dimension – Rank and nullity.

UNIT – III   Matrix and Inner product space

Matrix of a linear transformation – Inner product space – Definition and examples – Orthogonality – Gram Schmidt Orthogonalisation process – Orthogonal Complement.

UNIT – IV   Theory of Matrices:

Algebra of Matrices – Types of Matrices – The Inverse of a Matrix – Elementary Transformations – Rank of a Matrix.

UNIT – V   Characteristic equation and bilinear forms:

Characteristic equation and Cayley –Hamilton theorem – Eigen values and Eigen vectors.

Text book

1. Arumugam s and thangapandi Isaac A, Modern Algebra, sciTech publications (India) Ltd., Chennai, Edition 2012.

References

1. I. N. Herstein, Topics in Algebra, second Edition, John Wiley & sons (Asia), 1975.

LINEAR ALGEBRA

UNIT- I

CHOOSE THE CORRECT ANSWER

1. Hom (v,v) is a

a) Vector space

b) Subspace

c) Integer

d) None

2. Any two finite dimensional vector spaces over F of the same dimension are

a) Isomorphic

b) Non-isomorphic

c) Homomorphism

d) None

3. A & B are subspaces of V then $\frac{A+B}{B}$ is isomorphic to

a) A∩B/A

b) $A/A\cap B$

c) B/A∩B

d) None

4. The kernel of a homomorphism is a

a) Space

b) Homomorphism

c) Subspaces

d) None

5. Hom (v,w) is a

a) Ring

b) Subspace

c) Vector space

d) None

6. L(s) is a ………. of V

a) Subspace

b) Not a subspace

c) Dual space

d) None

7. Let V be a vector space, T is a linear transform on V into V ∋ Tα=0 ∀ α∈ V

a) T is identity transform

b) T is zero transform

c) T is invertible

d) T is orthogonal

8. The vector space which has only the additive identity element zero is

a) Complex space

b) Real space

c) Null space

d) None

9. If V and W are finite dimensional vector space over the same field. Then V and W are isomorphic iff

a) dim V $>$ dim W

b) dim V = dim W

c) dim V< dim W

d) None

10. If V = R^3 is a vector space and W = {(x, y,1); x, y∈ R} is a subset V then

a) W is a subspace of V

b) W is not a subspace of V

c) W is not a vector space

d) None of these

ANSWERS:

1) a 2) a 3) b 4) c 5) c 6) a 7) b 8) c 9) b 10) a

TWO MARK QUESTIONS

11. Define vector space.

12. Define subspace.

13. What is linear transformation?

14. What is trivial linear transformation?

15. Define direct sum.

16. What is linear combination?

17. Define linear span.

18. Define homomorphism.

19. Define quotient space.

20. Define kernel of homomorphism.

FIVE MARK QUESTIONS

21. Prove that R×R is a vector space over R under addition and scalar multiplication defined by $(x_1, x_2) + (y_1, y_2) = (x_1+y_1, x_2+y_2)$ and $\alpha (x_1, x_2) = (\alpha x_1, \alpha x_2)$.

22. Prove that C is a vector space over the field R.

23. Prove that the union of two subspaces of a vector space need not be a subspace.

24. R^n= {(x_1,x_2 ,..., x_n)/ x_i ∈R, 1≤i≤n} , then prove that R^n is a vector space over R under addition and scalar multiplication defined by

$(x_1 , x_{2,} \dots\dots . x_n) + (y_1, y_{2,} \dots\dots . y_n) = (x_1 + y_1, x_2 + y_2, \dots\dots, x_n + y_n)$

α (x_1, x_2,…… x_n)= (αx_1, αx_2,……, αx_n).

25. Let V= {a+b√2/a,b∈Q}. Then V is a vector space over Q under addition and multiplication.

26. Let V denote the set of all solution of the differential equation $2\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 3y = 0$. Then prove that V is a vector space over R.

27. In R^3, W = {(ka, kb, kc) / $k \in R$} is a subspace of R^3.

28. Let V be a vector space over a field F. A non empty subset W of V is a subspace of V iff u,v ∈ W and α,β ∈ F => αu + βv ∈ W.

29. T: $R^2 \rightarrow R^2$ defined by T(a,b) = (2a-3b , a+4b) is a linear transformation.

30. Let T: V→W be a linear transformation, then prove that T(V)={T(v) / v∈V} is a subspace of W.

TEN MARK QUESTIONS

31. Let V vector space over a field F. Then

(i) α0 = 0 ∀ α ∈F

(ii) 0v = 0 ∀ v ∈ V

(iii) (-α)v = α(-v) = -(αv) ∀ α∈F & v∈V

(iv) αv = 0 => α=0 (or) v=0.

32. Let V be a vector space over F. A non empty subset W of V is a subspace of V iff W is closed with respect to vector addition and scalar multiplication in V.

33. Prove that the intersection of two subspaces of a vector space is a subspace.

34. If A&B are subspaces of V, prove that A+B= {v∈V / v=a+b, a∈A, b∈B} is a subspace of V. Further show that A+B is the smallest subspace containing A&B.

35. Let A&B be subspace of a vector space V. Then A∩B={0} iff every vector v∈ A+B can be uniquely expressed in the form v= a+b where a∈A & b∈B.

36. State and prove fundamental theorem of homomorphism.

37. Let V be a vector space over a field F. Let A&B be subspace of V. Then prove that $\frac{A+B}{A} \cong \frac{B}{A\cap B}$.

38. Let V&W be a vector space over a field F. Let L(V,W) represent the set of all linear transform from V to W. Then L(V,W) itself is a vector space over F under addition and scalar multiplication defined by (f+g)(v) = f(v)+g(v) and (αf)(v) = αf(v).

39. Let V be a vector space over F and W a subspace of V. Let V/W = {W + v/ v∈V}. Then V/W is a vector space over F under the following operations.

(i) $(W+v_1) + (W+v_2) = W+v_1+v_2$

(ii) $\alpha(W+v_1) = W+\alpha v_1$

40.Let V be a vector space over a field F and S be a non- empty subset of V. Then prove that (i) L(S) is a subspace of V (ii) S ⊆ L(S) (iii) If W is any subspace of V such that S⊆ W, then L(S) ⊆ W. i.e L(S) is the smallest subspace of V containing S.

UNIT – II

CHOOSE THE CORRECT ANSWER

1. Let S be a subset of a vector space V a field F. S is called a basis of V if

a) S is linearly independent and L(S)=S
b) S is linearly independent and L(S)=V
c) S is linearly dependent and L(S)=V
d) S is linearly dependent and L(S)=S

2. A basis for the vector space consisting of all matrices of the form $\begin{pmatrix} a & b \\ 0 & d \end{pmatrix}$ Where a, b, d ∈ R is

a) $\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$
b) $\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right\}$
c) $\left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$
d) $\left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\}$

3. dim $M_2(R)$=

a) 1

b) 2

c) 3

d) 4

4. In $V_3(R)$, let S= L {(1, 1, 1)} and T = L {(-1, -1, -1)} .Then dim $(S \cap T)$ is

a) 1

b) 0

c) 2

d) 3

5. In $V_3(R)$, let S= L {(1, 1, 1)} and T = L {(-1, -1, -1)} .Then dim $(S+T)$ is

a) 1

b) 0

c) 2

d) 3

6. 1& i are linearly independent over the

a) Real

b) Complex

c) Integers

d) None

7. 1& i are linearly dependent over the

a) Real

b) Complex

c) Integers

d) None

8. dim (A+B)=

a) dim(A)+dim(B)-dim(A∩ B)

b) dim(A)-dim(B)

c) dim(A)+dim(B)+dim(A∩ B)

d) dim(A)+dim(B)

9. Any two finite dimensional vector space over F of the same direction are

a) Isomorphic

b) Non- isomorphic

c) Homomorphism

d) None of these

10. Hom (v,v)= n^2 then n is

a) Dimension of v

b) Number of v

c) Data inadequate

d) None of these

ANSWERS:

1)b 2) a 3) d 4) a 5) a 6) a 7) b 8) a 9) a 10) a

TWO MARK QUESTIONS

11. Define finite dimensional vector space.

12. Define linearly independent.

13. Define linearly dependent.

14. Define basis of a vector space.

15. Define standard basis.

16. Define maximal linearly independent set.

17. Define minimal generating set.

18. What is rank?

19. What is nullity?

20. Define singular and non-singular.

FIVE MARK QUESTIONS

21. Prove that any set containing a linearly dependent set is also linearly dependent.

22. Prove that any finite dimensional vector space V contains a finite of number linearly independent vectors which span V.

23. Prove that any two basis of a finite dimensional vector space V have the same number of elements.

24. In V_n (F), $\{e_1, e_2, \ldots, e_n\}$ is linearly independent set of vectors for $\alpha_1 e_1 + \alpha_2 e_2 + \ldots + \alpha_n e_n = 0$.

25. Let T: V→W be a linear transformation. Then prove that dim V = rank T + nullity T.

26. Let V be a finite dimensional vector space over a field F. Let A be a subspace of V. Then there exists a subspace B of V such that V= $A \oplus B$.

27. Let V be a vector space of dimension n. Then

    (i) Any set of m vectors where m>n is linearly dependent.

    (ii) Any set of m vectors where m<n cannot span V.

28. Prove that S= {(1,0,0), (0,1,0), (1,1,1)} is a basis for V_3(R) .

29. In V_3(R), the vectors (1,2,1),(2,1,0),&(1,-1,2) are linearly independent.

30. Let S= $\{v_1, v_2 \ldots v_n\}$ be a linearly independent set of vector in a vector space V over a field F. Then every element of L(S) can be uniquely written in the form $\alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n$ where $\alpha_i \in F$.

TEN MARK QUESTIONS

31. In $V_3(R)$ the vectors (1,4,-2) , (-2,1,3), (-4,11,5) are linearly dependent.

32. Any subset of a linearly independent set is linearly independent.

33. $S=\{ v_{1,} v_2,\ldots\ldots v_n \}$ is a linearly dependent set of vectors in V iff there exists a vector $v_k \in S$ such that v_k is a linear combination of the preceding vectors $v_{1,} v_2,\ldots\ldots v_{k-1.}$

34. Let V be a vector space over a field F. Then $S= \{ v_{1,} v_2,\ldots\ldots v_n \}$ is a basis for V iff every element of V can be uniquely expressed as a linear combination of element of S.

35. Let V be a vector space over a field F. Let $S=\{ v_{1,} v_2,\ldots\ldots v_n \}$ span V. Let $S=\{ w_{1,} w_2,\ldots\ldots w_n \}$ be a linearly independent set of vectors in V. Then $m \leq n$.

36. Let V be a finite dimensional vector space over a field F. Any linearly independent set of vectors in V is part of a basis.

37. Prove that any vector space of dimension n over a field F is isomorphic to $V_n(F)$.

38. Let V be a vector space over a field F. Let $S= \{ v_{1,} v_2,\ldots\ldots v_n \} \subseteq V$. Then the following are equivalent.
    (i) S is a basis for V.
    (ii) S is a maximal linearly independent set.
    (iii) S is a minimal generating set.

39. Let V be a finite dimensional vector space over a field F. Let w be a subspace of V.
    Then prove that (i) dim w/v = dim v
    (ii) dim v/w = dim v – dim w

40. Let V be a finite dimensional vector space over a field F. Let A&B be subspace of V. Then dim (A+B) = dim (A) + dim (B) – dim ($A\cap B$).

UNIT – III

CHOOSE THE CORRECT ANSWER

1. The standard inner product defined on $V_3(R)$ where $x=(x_{1,}x_{2,}x_3)$ & $(y_{1,}y_{2,}y_3)$ is

   a) $<x,y> = x_1y^2+ x_2y^2+x_3y^2$

   b) $<x,y> = x_1y_1+ x_2y_2+x_3y_3$

c) $<x,y> = x_1y + x_2y + x_3y$

d) $<x,y> = xy_1 + xy_2 + xy_3$

2. Let V be a vector space of polynomials with inner product defined by $<f, g> = \int_0^1 f(t)g(t)dt$. If f(t) = t & g(t) = t+1 then $<f,g>$ is

a) 1/6

b) 0

c) 5/6

d) -5/6

3. The orthogonal complement of inner product space V is

a) zero subspace {0}

b) V itself

c) Any subspace

d) None of these

4. If {0} is a zero subspace of inner product space V then $\{0\}^{\perp}$ is equal to

a) {0}

b) V

c) ϕ

d) None of these

5. The zero subspace of inner product space consists

a) Zero elements only

b) Non-zero elements

c) Identity

d) None of these

6. If V is an inner product space then for $\alpha, \beta \in V$ is

a) $\left|\left(\frac{\alpha}{\beta}\right)\right| = \|\alpha\|\|\beta\|$

b) $\left|\left(\frac{\alpha}{\beta}\right)\right| \geq \|\alpha\|\|\beta\|$

c) $\left|\left(\frac{\alpha}{\beta}\right)\right| \leq \|\alpha\|\|\beta\|$

d) None of these

7. If V is an inner product space then for $\alpha \in V$ is

a) $\|\alpha\| \leq 0\,, \alpha \neq 0$

b) $\|\alpha\| \geq 0\,, \alpha \neq 0$

c) $\|\alpha\| = 0\,, \alpha \neq 0$

d) $\|\alpha\| > 0\,, \alpha \neq 0$

8. If V is an inner product space $\alpha, \beta \in V$ is and c any scalar then

a) $\|c\alpha\| = \|c\|\|\alpha\|$

b) $\|c\alpha\| \geq \|c\|\|\alpha\|$

c) $\|c\alpha\| \leq \|c\|\|\alpha\|$

d) None of these

9. An orthogonal set of non-zero vectors

a) Linearly independent

b) Linearly dependent

c) constant

d) None of these

10. If T is a linear operator on a finite dimensional and space V and scalar C is a characteristics value of T then

a) $det(T - CI) \neq 0$

b) $det(T - CI) = 0$

c) $det(C - TI) > 0$

d) $det(T - CI) < 0$

ANSWERS

1)b 2)c 3)a 4)b 5)a 6)c 7)d 8)a 9)a 10)b

TWO MARK QUESTIONS

11. Define inner product space.
12. Define Euclidean space.
13. Define orthogonal.
14. Define orthogonal set.
15. What is orthonormal set?
16. State Gram – Schmidt orthogonalisation process.
17. Define orthogonal complement.
18. Define norm of inner product.
19. State Schwartz inequality.
20. State triangle inequality.

FIVE MARK QUESTIONS

21. Obtain the matrix representing the linear transformation T: $V_3(R) \rightarrow V_3(R)$ given by T(a,b,c) = (3a, a-b, 2a+b+c) with respect to the standard basis $\{e_1,e_2,e_3\}$.
22. Find the linear transformation T: $V_3(R) \rightarrow V_3(R)$ determined by the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ with respect to the standard basis $\{e_1,e_2,e_3\}$.
23. Prove that $V_n(R)$ is a real inner product space with inner product defined by $\langle x,y \rangle = x_1y_1 + x_2y_2 + \ldots\ldots + x_ny_n$ where x= $(x_1, x_2, \ldots\ldots x_n)$ & y= $(y_1, y_2, \ldots\ldots y_n)$.
24. Prove that $V_n(C)$ is a complex inner product space with inner product defined by $\langle x,y \rangle = x_1\overline{y_1} + x_2\overline{y_2} + \ldots\ldots + x_n\bar{y}_n$ where x= $(x_1, x_2, \ldots\ldots x_n)$ & y= $(y_1, y_2, \ldots\ldots y_n)$.

25. Let V be the set of all continuous real valued functions defined on the closed interval [0, 1]. Prove that V is a real inner product space with inner product defined by $\langle f, g\rangle = \int_0^1 f(t)g(t)dt$.

26. State and prove Schwartz's inequality.

27. State and prove Triangle inequality.

28. Let S={ $v_1, v_2, \ldots\ldots v_n$ } be an orthogonal set of non-zero vectors in an inner product space V. Then prove that S is linearly independent.

29. Let S={ $v_1, v_2, \ldots\ldots v_n$ } be an orthogonal set of non-zero vectors in V. Let v∈ V and $V = \alpha_1 v_1 + \alpha_2 v_2 + \ldots.. + \alpha_n v_n$. Then prove that $\alpha_k = \frac{\langle v, v_k\rangle}{\|v_k\|^2}$

30. Find a vector of unit length which is orthogonal to (1,3,4) in $V_3(R)$ with standard inner product.

TEN MARK QUESTIONS

31. Let V be the vector space of polynomials with inner product given by $\langle f, g\rangle = \int_0^1 f(t)g(t)dt$. Let f(t)=t+2 and g(t)=t^2-2t-3. Find (i) <f,g> (ii) $\|f\|$

32. The norm defined in an inner product space V has the following properties

(i) $\|x\| \geq 0$ and $\|x\| = 0$ iff x=0

(ii) $\|\alpha x\| = |\alpha|\|x\|$

(iii) $|\langle x, y\rangle| \leq \|x\|\|y\|$

(iv) $\|x + y\| \leq \|x\| + \|y\|$

33. Every finite dimensional inner product space has an orthonormal basis.

34. Apply Gram-Schmidt process to construct an orthonormal basis for $V_3(R)$ with the standard inner product for the basis {v_1, v_2, v_3} where v_1=(1,0,1), v_2=(1,3,1) and v_3= (3,2,1).

35. Find an orthogonal to (1,3,4) are the points lying on the plane x+3y+4z=0 which is a two dimensional subspace of $V_3(R)$.

36. Let V be the set of all polynomials of degree ≤ 2 together with zero polynomial. V is a real inner product space with inner product defined by $\langle f, g\rangle = \int_{-1}^1 f(x)g(x)dx$ Stating with the basis {1,x,x^2 }. Obtain an orthonormal basis for V.

37. Let V be a finite dimensional inner product space. Let W be a subspace of V. Then prove that V is the direct sum of W and $W^{\perp}$. i.e., V= W $\oplus$ $W^{\perp}$.

38. (i) If S is any subset of V then prove that $S^{\perp}$ is a subspace of V.
(ii) Let V be a finite dimensional inner product space. Let W be a subspace of V. Then prove that $(W^{\perp})^{\perp}$ = W.

39. Let W_1 & W_2 be subspace of a finite dimensional inner product space. Then Prove that (i) $(W_1 + W_2)^{\perp} = W_1^{\perp} \cap W_2^{\perp}$ and (ii) $(W_1 \cap W_2)^{\perp} = W_1^{\perp} + W_2^{\perp}$

40. Let V & W be two finite dimensional vector space over a field F. Let dim V=m & dim W = n. Then L (V,W) is a vector space of dimension mn over F.

UNIT – IV

CHOOSE THE CORRECT ANSWER

1. If A = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ then A^T is

a) $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

d) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2. If A is any 3×2 matrix then $(A^T)^T$ is

a) 3×3 matrix

b) 3×2 matrix

c) 2×2 matrix

d) 2×3 matrix

3. A & B are m×n matrices. Then the order of A^T+B^T is

a) m×n

b) m×m

c) n×n

d) n×m

4. If A= $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ then the conjugate of A is

a) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 9 \\ 4 & 3 \end{pmatrix}$

d) $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$

5. Conjugate of A = $\begin{pmatrix} 1+i & 2 \\ 2-i & i \end{pmatrix}$ is

a) $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

b) $\begin{pmatrix} 1-i & 2 \\ 2+i & -i \end{pmatrix}$

c) $\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$

d) $\begin{pmatrix} 1 & 1 \\ i & 1 \end{pmatrix}$

6. If the elements of a 2×2 matrix are given by the formula a_{ij}=i+j+ij, then the first element in the first row is

a) 1

b) 2

c) 3

d) 4

7. A square matrix A is said to be idempotent if

a) A^2=0

b) $A^2=A$

c) $A^2=I$

d) $A=A^{-1}$

8. A square matrix A is said to be involutory if

a) $A^2=0$

b) $A^2=A$

c) $A^2=I$

d) $A=A^{-1}$

9. If A= $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ then adj A is

a) $\begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$

b) $\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$

c) $\begin{pmatrix} 3 & 6 \\ 4 & 5 \end{pmatrix}$

d) $\begin{pmatrix} 1 & 1 \\ i & 1 \end{pmatrix}$

10. If A= $\begin{pmatrix} 1 & 2 \\ 3 & n \end{pmatrix}$ & B= $\begin{pmatrix} m & 2 \\ n & 4 \end{pmatrix}$ are singular matrices then the value of mn is

a) 1

b) 3

c) 2

d) 4

ANSWERS

1)d 2)b 3)d 4)b 5)b 6)c 7)b 8)c 9)b 10)b

TWO MARK QUESTIONS

11. Define singular and non-singular.

12. Define symmetric matrix.

13. Define skew-symmetric matrix.

14. Define orthogonal matrix.

15. Define hermitian matrix.

16. Define involutory.

17. If A= $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ then find A^{-1}.

18. If A= $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then find A^{-1}.

19. If A= $\begin{pmatrix} 1+i & 1-i \\ i & -1 \end{pmatrix}$ then find A+ A^{-1}.

20. Define unitary matrix.

FIVE MARK QUESTIONS

21. Find the value of $(2 \quad 1 \quad -1)\begin{pmatrix} 4 & -1 & 2 \\ 0 & -1 & 1 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix}$

22. Find the value of x satisfying the equation $(x \quad 1)\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} x \\ 2 \end{pmatrix}$=0

23. If A= $\begin{pmatrix} 1 & 0 & 0 \\ 2 & x & 4 \\ 4 & 1 & x \end{pmatrix}$ is a singular matrix then find the value of x.

24. If A= $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ then find A^{-1}.

25. If A= $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ then find the cofactor matrix (A_{ij}).

26. Prove that a square matrix A is symmetric iff $A=A^T$.

27. Let A&B be orthogonal matrices of the same order. Then find
    (i) A^T is orthogonal

    (ii) AB is orthogonal.

28. Prove that a square matrix A of order n is non-singular iff A is invertible.

29. Let A be an m× n matrix, B be an n× p matrix and c be a p×q matrix. Then find A(BC)=(AB)C.

30. Let A be an m× n matrix, B be an n× p matrix. Then find $(AB)^T=B^TA^T$.

TEN MARK QUESTIONS

31. Let A and B be symmetric matrices of order n. Then find

(i) A+B is symmetric

(ii) AB is symmetric iff AB=BA

(iii) AB+BA is symmetric

(iv) If A is symmetric then kA is symmetric where k∈F.

32. Let A and B be skew- symmetric matrices of order n. Then find

(i) A+B is skew-symmetric

(ii) kA is skew-symmetric , k∈F

(iii) A^{2n} is a symmetric and A^{2n+1} is skew- symmetric matrix where n is any positive integer.

33. Let A&B be square matrices of the same order. Then find

(i) A,B are hermitian => A+B is hermitian

(ii) A is hermitian => i A is skew hermitian

(iii) A,B are hermitian => AB+BA is hermitian.

34. Compute the inverse of the matrix A= $\begin{pmatrix} 2 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$

35. If $\omega = e^{2\pi i/3}$ find the inverse of the matrix A =$\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}$

36. Reduce the matrix A= $\begin{pmatrix} 1 & 2 & -1 \\ 1 & 1 & 2 \\ 2 & 4 & -2 \end{pmatrix}$ to the canonical form.

37. Find the inverse of the matrix A = $\begin{pmatrix} 1 & 0 & 2 \\ 3 & 1 & -1 \\ -2 & 1 & 3 \end{pmatrix}$

38. Find the rank of the matrix A=$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 4 & 2 \end{pmatrix}$ by examining the determinant minors.

39. Find the rank of the matrix A= $\begin{pmatrix} 3 & -1 & 2 \\ 0 & 1 & -3 \\ 6 & -1 & 1 \end{pmatrix}$

40. Prove that the row rank and the column rank of any matrix are equal.

UNIT- V

CHOOSE THE CORRECT ANSWER

1. The characteristic polynomial of A = $\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ is

a) x^2-2x+5

b) x^2+2x+5

c) x^2-2x-5

d) x^2+2x-5

2. The characteristic polynomial of A = $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ is

a) x^2

b) x^2-1

c) 1-x^2

d) x-1

3. The characteristic equation of $\begin{pmatrix} -m & -n \\ 1 & 0 \end{pmatrix}$ is

a) x^2-mx-n = 0

b) x^2+mx+n = 0

c) x^2+nx+m = 0

d) x^2+nx+mn = 0

4. The characteristic equation of A = $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ is

a) $x^2-2x+1 = 0$

b) $x^2+2x+1 = 0$

c) $x^2+x+2 = 0$

d) $x^2-x-1 = 0$

5. The Eigen values of the matrix I_2 are

a) -1, 1

b) -1, -1

c) 1, -1

d) 1, 1

6. If the Eigen values of a square matrix A are 1, 2, 3 then the eigen values of A^2 are

a) 1, 4, 9

b) 1, 2, 3

c) -1, -4, -9

d) 1, 3, 5

7. The characteristic roots of $\begin{pmatrix} cos\theta & -sin\theta \\ -sin\theta & cos\theta \end{pmatrix}$ are

a) -1, -1

b) 1, 1

c) $\cos^2\theta$ - $\sin^2\theta$

d) $\cos\theta + \sin\theta$

8. If the Eigen values of A = $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ are -2, 3, 6 then the Eigen values of A^T are

a) -2, 3, 6

b) $-\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$

c) $-2^2, 3^2, 6^2$

d) -4, 6, 12

9. The sum of the Eigen values of $\begin{pmatrix} cos\theta & -sin\theta \\ -sin\theta & -cos\theta \end{pmatrix}$ is

a) 0

b) 1

c) 2cosθ

d) cos$^2\theta$

10. The sum and product of the Eigen values of $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ are

a) 0, 0

b) 3, 3

c) 3, 1

d) 2, 1

ANSWERS

1)a 2)a 3)b 4)a 5)d 6)a 7)c 8)a 9)a 10)c.

TWO MARK QUESTIONS

11. Define Characteristic matrix.

12. Define Characteristic polynomial.

13. Define characteristic equation.

14. State Cayley Hamilton theorem.

15. Define Eigen value.

16. Define Eigen vector.

17. Define Characteristic roots.

18. If A = $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ then find the sum and product of the Eigen values of A.

19. Find the Eigen value of $\begin{pmatrix} 8 & -6 & 6 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

20. Find the Eigen value of A =$\begin{pmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{pmatrix}$.

FIVE MARK QUESTIONS

21. Find the characteristic equation of A =$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.

22. Find the characteristic equation of A =$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{pmatrix}$,

23. Find the characteristic equation of the matrix A = $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$

24. Show that the matrix A = $\begin{pmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{pmatrix}$ satisfies A(A-I)(A+2I) = 0.

25. Verify Caley Hamilton's theorem for the matrix A = $\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$.

26. Prove that the Eigen values of A and its transpose A^T are the same.

27. If λ is an Eigen value of a non singular matrix A then $\frac{1}{\lambda}$ is an eigen value of A^{-1}

28. If λ is an Eigen value of A then kλ is an Eigen value of KA where K is a scalar.

29. Verify the statement that the sum of the elements in the diagonal of a matrix is the sum of the Eigen values of the matrix A =$\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$.

30. Find the sum of the squares of the Eigen values of A =$\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$.

TEN MARK QUESTIONS

31. State and prove Cayley Hamilton theorem.

32. Using cayley Hamilton theorem, find the inverse of the matrix $\begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix}$.

33. Using cayley Hamilton theorem for the matrix A = $\begin{pmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{pmatrix}$, find A^{-1} & A^4.

34. Let A be a square matrix. Then find
    (i) The sum of the Eigen values of A is equal to the sum of the diagonal elements of A.
    (ii) Product of Eigen values of A is $|A|$.

35. Prove that Eigen vectors corresponding to distinct Eigen values of a matrix are linearly independent.

36. Prove that the characteristic roots of a Hermitian matrix are all real.

37. The product of two Eigen values of the matrix A= $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ is 16. Find the 3rd Eigen value. What is the sum of the Eigen values of A?

38. Find the Eigen values and Eigen vectors of the matrix A= $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$.

39. Find the Eigen values and Eigen vectors of the matrix A= $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$.

40. Find the Eigen values and Eigen vectors of the matrix A= $\begin{pmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$.