ஸ்ரீ-ல-ஸ்ரீ காசிவாசி சுவாமிநாத சுவாமிகள் கலைக் கல்லூரி

திருப்பனந்தாள் – 612504

S.K.S.S ARTS COLLEGE, THIRUPPANANDAL - 612504

QUESTION BANK

Title of the Paper

CALCULUS AND FOURIER SERIES

COURSE – I PHYSICS

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ALLIED COURSE I

CALCULUS AND FOURIER SERIES

Objects :

1. To learn the basic need for their major concepts
2. To train the students in the basic Integrations

UNIT I :

Successive Differentiation – n^{th} derivative of standard functions (Derivation not needed) simple problems only-Leibnitz Theorem (proof not needed) and its applications- Curvature and radius of curvature in Cartesian only (proof not needed)–Total differential coefficients (proof not needed) - Jacobians of two & three variables –Simple problems in all these.

UNIT II :

Evaluation of integrals of types

1] $\int \frac{px+q}{ax^2+bx+c}\,dx$ 2] $\int \frac{px+q}{\sqrt{ax^2+bx+c}}\,dx$ 3] $\int \frac{px+q}{(x+p)\sqrt{ax^2+bx+c}}\,dx$

4] $\int \frac{dx}{a+bcosx}$ 5] $\int \frac{dx}{a+bsinx}$ 6] $\int \frac{(acosx+bsinx+c)}{(pcosx+qsinx+r)}\,dx$

Integration by trigonometric substitution and by parts of the integrals

1] $\int \sqrt{a^2-x^2}\,dx$ 2] $\int \sqrt{a^2+x^2}\,dx$ 3] $\int \sqrt{x^2-a^2}\,dx$

UNIT III :

General properties of definite integrals – Evaluation of definite integrals of types

1] $\int_a^b \frac{dx}{\sqrt{(x-a)(b-x)}}$ 2] $\int_a^b \sqrt{(x-a)(b-x)}\,dx$ 3] $\int_a^b \sqrt{\frac{x-a}{b-x}}\,dx$

Reduction formula (When n is a positive integer) for

1] $\int_a^b e^{ax}x^n dx$ 2] $\int_a^b \sin^n x\,dx$ 3] $\int_a^b \cos^n x\,dx$

4] $\int_a^b e^{ax}x^n dx$ 5] $\int_a^{\frac{\pi}{2}} \sin^n x\,dx$ 6] without Proof $\int_a^{\frac{\pi}{2}} \sin^n x\cos^m x\,dx$ - and illustrations

UNIT IV :

Evaluation of Double and Triple integrals in simple cases –Changing the order and evaluating of the double integration. (Cartesian only).

UNIT V :

Definition of Fourier Series – Finding Fourier Coefficients for a given periodic function with period 2π and with period $2l$ - Use of Odd & Even functions in evaluating Fourier Coefficients - Half range sine & cosine series.

TEXT BOOK(S) :

1. S. Narayanan, T.K. Manichavasagam Pillai, Calculus, Vol. I, S. Viswanathan Pvt Limited, 2003
2. S. Arumugam, Isaac and Somasundaram, Trigonometry & Fourier Series, New Gamma Publishers, Hosur, 1999.

UNIT - I

Choose the correct Answer :

1) If y=x find the $\frac{d^3y}{dx^3}$

   a) 0

   b) 1

   c) x

   d) -1

2) If y=x^4 find the $\frac{d^4y}{dx^4}$

   a) 4

   b) $12x^2$

   c) $24x$

   d) 24

3) If $2ay = x(b + a\ dy/dx)$, then $y_3 =$ ....

   a) Constant

   b) 0

   c) $a + b$

   d) None of these

4) If $\sqrt{x} + \sqrt{y} = 5$, then $y_2 =$?

   a) $\frac{\sqrt{a}}{2x^{3/2}}$

   b) $\frac{25}{2x^{3/2}}$

   c) $\frac{5}{2x^{3/2}}$

d) $\frac{1}{10}$

5) If $y = (2\, tan^{-1}x)^2$, then.

a) $(x^2+1)^2 y_2 + 2x(x^2+1)\, y_1 + 8 = 0$

b) $(x^2+1)^2 y_2 + 2x(x^2+1)\, y_1 = 8$

c) $(x^2-1)^2 y_2 + 2x(x^2-1)\, y_1 = 2$

d) $(x^2+1)^2 y_1 + 2x(x^2+1)\, y_2 + 2 = 0$

6) If $f(x) = \begin{cases} x^2\, sin 1/x, & x \neq 0 \\ 0, & x = 0 \end{cases}$ then

a) f'(x) is continuous at x=0

b) f'(x) is differentiable at x=0

c) f'(x) is not differentiable at x=0

d) None of these

7) If $y = sin(ax+b)$, then

a) $y_n = a^n \cos(ax + b + \frac{n\pi}{2})$

b) $y_n = b^n \sin(ax + b + \frac{n\pi}{2})$

c) $y_n = a^n \sin(ax + b + \frac{n\pi}{2})$

d) $y_n = \sin\left(ax + b + \frac{n\pi}{2}\right)$

8) If $y = e^x \sin(ax+b)$, then.

a) $y_{20} = (1+a^2)^{10} e^x \sin(ax + 20tan^{-1}a)$

b) $y_{20} = (1+a^2)^{10} \sin(ax + b + 20tan^{-1}a)$

c) $y_{20} = (1+a\,)^{10} \cos(ax + b + 20tan^{-1}a)$

d) $y_{20} = (1+a^2)^{10} e^x \sin(ax + b + 20tan^{-1}a)$

9) If $y = \frac{x}{x^2-1}$,then y_{99} =?

a) $-99!$

b) $99!$

c) 99

d) 0

10) If u and v are two function of x having derivatives of n^th order, then

$(uv)_n = n_{c_0} u_n v + n_{c_1} u_{n-1} v_1 + \cdots + n_{c_r} u_{n-r} v_r + \cdots + n_{c_n} u\, v_n$

a) Cauchy's theorem

b) Lagrange's theorem

c) Leibnitz's theorem

d) Lipchitz's theorem

Answers :

1) a 2) d 3) b 4) c 5) b 6) c 7) c 8) d 9) a 10) c

2 Marks :

11) If $y = acos2x + bsin3x$ find $\frac{d^2y}{dx^2}$.

12) If $y = 2sinx + 3cosx$ find $\frac{d^2y}{dx^2}$.

13) To find the n^{th} derivative of e^{ax}

14) If $y = e^{5x}$ find y_n .

15) If $y = ae^{mx} + be^{-mx}$ show that $\frac{d^2y}{dx^2} - m^2y = 0$

16) If $y = e^{2x}$ find y_{10} .

17) If $y = e^{-3x}$ find y_6 .

18) Find $D^n[\sin(2x+2)]$

19) Find $D^n[\cos(2-5x)]$

20) Find $D^n[\sin^2 x]$

5 Marks :

21) If $y=\frac{x+1}{x+2}$ find $\frac{d^2y}{dx^2}$.

22) If $y=\frac{x+2}{x+3}$ find $\frac{d^2y}{dx^2}$.

23) If $y=(sin^{-1}x)^2$ prove that $(1-x^2)(y_2-xy)=2$.

24) If $y=e^{tan^{-1}x}$ show that $(1+x^2)y_2+(2x-1)y_1=0$

25) To find the n^th^ derivative of $\frac{1}{(ax+b)^2}$.

26) Find $D^n[\cos^3 x]$

27) Find the n^th^ derivative of $e^{3x}\, sin4x$.

28) To find the n^th^ derivative of $\frac{1}{ax+b}$.

29) To find the n^th^ derivation of $cos(ax+b)$.

30) Find $D^n[\log(4-x^2)]$.

10 Marks :

31) If $x=a(cos\theta+\theta sin\theta)$ and $y=a(sin\theta+\theta cos\theta)$ find $\frac{d^2y}{dx^2}$.

32) If $x=\sqrt{sin\ 2t}$ and $y=\sqrt{cos\ 2t}$ find $\frac{dy}{dx}$.

33) If $x=a(t-sint)$ and $y=a(1+cost)$ find $\frac{d^2y}{dx^2}$.

34) Formation of equation involving derivations. If $y=a\cos\ (log\ x)+b\sin(logx)$ show that $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$.

35) If $y=\sin(sinx)$ then prove that $\frac{d^2y}{dx^2}\ tanx+y\ cos^2x=0$

36) To find the n^th^ derivation of $\log(ax+b)$.

37) To find the nth derivation of $sin(ax+b)$.

38) Find $D^n\left[\frac{1}{(x+1)(x+3)}\right]$

39) Find $D^n\left[\frac{3x-1}{x(x+1)(x-1)}\right]$

40) Find the nth derivation of $\left[\frac{x^2-4}{(x+1)(x+4)}\right]$.

UNIT - II

Choose the correct Answer :

1) If $\emptyset(x)=\int_{x^2}^{0}\sqrt{t}\,dt$, then $\frac{d\emptyset}{dx}$

   a) $2x^2$

   b) $\sqrt{x}$

   c) 0

   d) 1

2) The value of $a=\int_0^{5p}(2-sinx)dx$

   a) >0

   b) 2

   c) 0-1+100-10+1

   d) Undefined

3) The value of the improper integral $\int_0^1 x\ln x$

   a) 1/4

   b) 0

   c) -1/4

   d) 1

4) Value of the definite integral $\int_{-\pi/2}^{\pi/2}\frac{sin2x}{1+cosx}\,dx$ is ……….

   a) $-2ln2$

   b) 2

   c) 0

   d) $(ln2)^2$

5) The value of integral $\int_{-2}^{2} \frac{dx}{x^2}$ is …………..

a) 0

b) 0.25

c) 1

d) ∞

6) $\int \frac{sinx}{1+sinx}\, dx = ?$

a) $secx + tanx + x + c$

b) $secx - tanx + x + c$

c) $-secx + tanx + x + c$

d) $-secx - tanx + x + c$

7) $\int cos4x\, cosx\, dx = ?$

a) $\frac{1}{5}\ sin5x + \frac{1}{3}\ sin3x + c$

b) $\frac{1}{5}\ cos5x - \frac{1}{3}\ cos3x + c$

c) $\frac{1}{10}\ sin5x + \frac{1}{6}\ sin3x + c$

d) None of these

8) In $\int 2^{3x+4}\, dx = ?$

a) $\frac{3}{log2}\ 2^{3x+4} + c$

b) $\frac{2^{3x+4}}{3(log2)} + c$

c) $\frac{2^{3x+4}}{2(log3)} + c$

d) None of these

9) $\int \sin(logx)\, dx = ?$

a) $\frac{1}{2} x \sin(logx) + \frac{1}{2} x \cos(logx) + c$

b) $\frac{1}{2} x \sin(logx) - \frac{1}{2} x \cos(logx) + c$

c) $-\frac{1}{2} x \sin(logx) + \frac{1}{2} x \cos(logx) + c$

d) None of these

10) Poisson $\int \frac{1}{\sqrt{sin^3 x cosx}}\, dx = ?$

a) $2\sqrt{tanx} + c$

b) $2\sqrt{cotx} + c$

c) $-2\sqrt{tanx} + c$

d) $-\frac{2}{\sqrt{tanx}} + c$

Answers :

1) a 2) a 3) c 4) d 5) c 6) b 7) c 8) b 9) b 10) d

2Marks :

11) Evaluate $\int \frac{2x+5}{\sqrt{x^2+3x+1}} \, dx$

12) Evaluate the integral $I = \int \frac{6x^2+2cosx}{x^3+sinx} \, dx$

13) Evaluate $\int \frac{1}{x^2+x} \, dx$

14) Evaluate the integral $I = \int \frac{1}{\sqrt{1-x^2}} \, dx$

15) Evaluate$\int \frac{2}{1+3cosx} \, dx$

16) Evaluate the integral $I = \int \frac{1}{x^2+4x+7} \, dx$

17) Let $\int \frac{2x-1}{\sqrt{x^2+7x+3}} \, dx$ Find the value of A and B

18) Evaluate the integral $I = \int x^2\left(xe^{-x^2}\right) \, dx$

19) Evaluate $\int e^{ax} cosbx \, dx$

20) Let $\int \frac{2x+3}{x^2+5x+7} \, dx$ To find A and B value

5 Marks :

21) Evaluate $\int \frac{3x+5}{x^2+4x+7} \, dx$

22) Evaluate $\int \frac{2x+3}{x^2+2x+3} \, dx$

23) Evaluate $\int \frac{dx}{13+12cosx}$

24) Evaluate $\int \frac{dx}{7-5\,sinx}$

25) Evaluate $\int \sqrt{a^2+x^2} \, dx$

26) Evaluate $\int \frac{dx}{2.3cosx}$

27) Evaluate $\int \sqrt{3+2x-x^2} \, dx$

28) Evaluate $\int \sqrt{x^2+4x+1} \, dx$

29) Evaluate $\int \sqrt{x^2-4x+6} \, dx$

30) Evaluate $\int \frac{dx}{5+4sinx}$

10 Marks :

31) Evaluate $\int \frac{2x+3}{x^2+5x+7}\, dx$

32) Evaluate $\int \frac{5x+1}{x^2-2x-35}\, dx$

33) Evaluate $\int \frac{2x-1}{\sqrt{x^2+7x+3}}\, dx$

34) Evaluate $\int \frac{x+4}{\sqrt{x^2+3x+4}}\, dx$

35) Evaluate $\int \frac{dx}{(x-1)\sqrt{5x^2-2-2x^2}}$

36) Evaluate $\int \frac{8cosx+sinx+5}{3cosx+2sinx+4}\, dx$

37) Evaluate $\int \frac{3+2sinx+cosx}{5+8cosx}\, dx.$

38) Evaluate $\int \sqrt{a^2-x^2}\, dx$

39) Evaluate $\int \sqrt{x^2-a^2}\, dx$

40) Evaluate $\int \frac{dx}{8+9\, cosx}$

UNIT - III

Choose the correct Answer :

1) $\int \frac{1}{x\, cos^2(1+logx)}\, dx = ?$

a) $tan(1+logx)+c$

b) $cot(1+logx)+c$

c) $sec(1+logx)+c$

d) None of these

2) $\int \frac{1}{\sqrt{x+3}-\sqrt{x+2}}\, dx = ?$

a) $\frac{2}{3}(x+3)^{3/2}-\frac{2}{3}(x+2)^{3/2}+c$

b) $\frac{2}{3}(x+3)^{3/2}+\frac{2}{3}(x+2)^{3/2}+c$

c) $\frac{3}{2}(x+3)^{3/2}-\frac{3}{2}(x+2)^{3/2}+c$

d) $\frac{3}{2}(x+3)^{3/2}+\frac{3}{2}(x+2)^{3/2}+c$

3) $\int \sin^3(2x+1)\, dx = ?$

a) $\frac{1}{8}sin^4(2x+1)+c$

b) $\frac{1}{2}cos\ (2x+1)+\frac{1}{3}cos^3(2x+1)+c$

c) $-\frac{1}{2}cos\ (2x+1)+\frac{1}{6}cos^3(2x+1)+c$

d) None of these

4) $\int\sqrt{e^x-1}\ dx=?$

a) $\frac{2}{3}(e^x-1)^{3/2}+c$

b) $\frac{1}{2}\frac{e^x}{\sqrt{e^x-1}}+c$

c) $2\sqrt{e^x-1}-2tan^{-1}\sqrt{e^x-1}+c$

d) None of these

5) $\int\frac{1}{\sqrt{x-x^2}}\ dx=?$

a) $sin^{-1}(x-1)+c$

b) $sin^{-1}(x+1)+c$

c) $sin^{-1}(2x-1)+c$

d) $sin^{-1}(2x+1)+c$

6) $\int\frac{x+3}{(x+4)^2}e^x\ dx=?$

a) $\frac{e^x}{x+4}+c$

b) $\frac{e^x}{x+3}+c$

c) $\frac{1}{(x+4)^2}+c$

d) $\frac{1}{(x+3)^2}+c$

7) $\int 7\sin(x)\,dx=?$

a) $7\ cos(x)+c$

b) $-7\ cos(x)+c$

c) $7\ sin(x)+c$

d) $-7\ sin(x)+c$

8) $\int 9\sin(3x)\,dx=?$

a) $\int -3\cos(3x)+c$

b) $\int 3\cos(3x)+c$

c) $\int 6\cos(3x)+c$

d) $\int -6\cos(3x)+c$

9) $\int 7\cos(5x)\,dx$ =?

a) $\frac{5\sin(5x)}{7} + c$

b) $\frac{\sin(5x)}{5} + c$

c) $\frac{\sin(7x)}{7} + c$

d) $\frac{7\sin(5x)}{5} + c$

10) $\int 4e^{-7x}dx$ =?

a) $-\frac{4e^{-7x}}{4} + c$

b) $\frac{4e^{-7x}}{7} + c$

c) $-\frac{4e^{-7x}}{7} + c$

d) $-\frac{e^{-7x}}{4} + c$

Answers :

1) a 2) b 3) c 4) b 5) c 6) a 7) b 8) a 9) d 10) c

2 Marks :

11) If f(x) is an even function of x, then $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$.

12) Evaluate $\int \sqrt{(x-3)(7-x)}\ dx$.

13) Evaluate $\int \frac{dx}{\sqrt{(x-1)(2-x)}}$

14) Evaluate $\int \sqrt{\frac{x-1}{2-x}}\ dx$.

15) Evaluate $\int_0^{\pi/2} sin^{10}x\ dx$.

16) Evaluate $\int_0^{\pi/2} sin^{9}x\ dx$.

17) Evaluate $\int_0^{\pi/2} cos^{9}x\ dx$.

18) Evaluate $\int_0^{\pi/2} cos^{8}x\ dx$.

19) Reduction formula for $\int_0^{\pi/2} sin^4x.cos^6x\ dx$.

20) Reduction formula for $\int_0^{\pi/2} sin^5x.cos^6x\ dx$.

5 Marks :

21) If f(x) is an odd function of x then $\int_{-a}^{a} f(x)dx = 0$.

22) Evaluate $\int \frac{dx}{\sqrt{(x-a)(b-x)}}$.

23) Reduction formula for $\int x^n e^{ax}\, dx$

24) Evaluate $\int_0^{3\pi/2} cos^5(\theta/3)\, d\theta$

25) Evaluate $\int_0^{\pi/6} sin^8(3\theta)\, d\theta$

26) Evaluate $\int_0^{2\pi} sin^7(x/4)\, dx$

27) Evaluate $\int_0^{\frac{\pi}{2}} cos^{14}\theta\, d\theta$

28) Reduction formula for $\int sin^m x. cos^n x\, dx$.

29) Reduction formula for $\int_0^{\pi/2} sin^8 x. cos^6 x\, dx$.

30) Reduction formula for $\int_0^{\pi/2} sin^9 x. cos^5 x\, dx$.

10 Marks :

31) Evaluate $\int_a^b \sqrt{(x-a)(b-x)}\, dx$. $a < b$.

32) Evaluate $\int \sqrt{(x-a)(b-x)}\; dx$.

33) Evaluate $\int_a^b \frac{dx}{\sqrt{(x-a)(b-x)}}$.

34) Evaluate $\int_a^b \sqrt{\frac{x-a}{b-x}}\; dx$

35) Evaluate $\int \sqrt{\frac{x-a}{b-x}}\; dx$

36) Reduction formula for $\int sin^n x\, dx$

37) Reduction formula for $\int_0^{\pi/2} sin^n x\, dx$

38) Reduction formula for $\int cos^n x\, dx$

39) Reduction formula for $\int_0^{\pi/2} cos^n x\, dx$

40) Evaluate $\int x^5 e^{2x} dx$.

UNIT - IV

Choose the correct Answer :

1) Find the value of $\int\int xy\, e^{x+y} dx\, dy$.

   a) $ye^{y}(xe^{x}-e^{x})$

   b) $(ye^{y}-e^{y})(xe^{x}-e^{x})$

   c) $(ye^{y}-e^{y})\, xe^{x}$

   d) $(ye^{y}+e^{y})(xe^{x}+e^{x})$

2) Find the value of $\int\int \frac{x}{x^2+y^2} dx\, dy$.

   a) $[y\, tan^{(-1)}(y)-\frac{1}{2}\ln(1+y^2)]$

   b) $x[y\, tan^{(-1)}(x)+\frac{1}{2}\ln(1+y^2)]$

   c) $y[y\, tan^{(-1)}(x)-\frac{1}{2}\ln(1+x^2)]$

   d) $x[y\, tan^{(-1)}(y)-\frac{1}{2}\ln(1+y^2)]$

3) Find the integration of $\int\int_0^x x^2+y^2\, dx\, dy$.

   a) $x^4/6$

   b) y

   c) $2x^3/3y$

   d) 1

4) Find the area inside function $(2x^3+5x^2-4)/x^2$ from $x=1$ to a.

   a) $\frac{a^2}{2}+5a-4\ln(a)$

   b) $\frac{a^2}{2}+5a-4\ln(a)-\frac{11}{2}$

   c) $\frac{a^2}{2}+5a-\frac{11}{2}$

   d) $\frac{a^2}{2}+4\ln(a)-\frac{11}{2}$

5) The mean value of a function f(x) from a to b is given by

   a) $\frac{f(a)+f(b)}{2}$

   b) $\frac{f(a)+2f\left(\frac{a+b}{2}\right)+f(b)}{4}$

   c) $\int_a^b f(x)\, dx$

d) $\frac{\int_a^b f(x)\,dx}{b-a}$

6) The exact value of $\int_{0.2}^{2.2} x\, e^x\, dx$ is most nearly

a) 7.8036

b) 11.807

c) 14.034

d) 19.611

7) $\int_{0.2}^{2} f(x)\,dx$ for $f(x) = \begin{cases} x & 0 \le x \le 1.2 \\ x^2 & 1.2 < x \le 2.4 \end{cases}$ is most nearly

a) 1.9800

b) 2.6640

c) 2.7907

d) 4.7520

8) The area of a circle of radius a can be found by the following integral.

a) $\int_0^a (a^2 - x^2)\,dx$

b) $\int_0^{2\pi} \sqrt{a^2 - x^2}\,dx$

c) $4\int_0^a \sqrt{a^2 - x^2}\,dx$

d) $\int_0^a \sqrt{a^2 - x^2}\,dx$

9) $\int sec^2\, 5x\, dx = ?$

a) $tanx + c$

b) $\frac{1}{5}\, tanx + c$

c) $\frac{1}{5}\, tan5x + c$

d) $tan5x + c$

10) $\int cos\sqrt{x}\, dx = ?$

a) $\sqrt{x}\, sin\sqrt{x}$

b) $2\sqrt{x}\, sin\sqrt{x}$

c) $\frac{1}{\sqrt{x}}\, sin\sqrt{x}$

d) $\frac{1}{2\sqrt{x}}\, sin\sqrt{x}$

Answers :

1) b 2) d 3) c 4) b 5) d 6) b 7) c 8) c 9) c 10) b

2 Marks :

11) Evaluate $\int_0^1 \int_0^1 dx\, dy$

12) Evaluate $\int_1^2 \int_1^2 x^2 y^2\, dx\, dy$.

13) Evaluate $\int_2^3 \int_2^1 xy\, dx\, dy$.

14) Evaluate $\int_0^1 \int_0^x x^2\, dx\, dy$.

15) Evaluate the double integral $\int_0^1 \int_0^{x^2} (x^2 + y^2)\, dy\, dx$.

16) Evaluate $\int_1^2 \int_2^3 x\, dx\, dy$.

17) Write the properties of Double integral

18) Write the properties of Triple integral

19) Define Area of a Double integral

20) Evaluate $\int_0^1 \int_0^1 \int_0^1 (x + y + z)\, dx\, dy\, dz$.

5 Marks :

21) Evaluate $\int_0^1 \int_1^2 (x^2 + y^2)\, dx\, dy$.

22) Evaluate $\int_0^3 \int_0^2 \int_0^1 xyz\, dx\, dy\, dz$

23) Evaluate $\int_0^1 \int_0^1 \int_0^1 xyz\, dx\, dy\, dz$

24) Evaluate $\int_0^2 \int_0^3 \int_0^1 (x + y - z^2)\, dx\, dy\, dz$.

25) Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dz\, dy\, dx$.

26) Evaluate $\int \int \int \frac{dx\, dy\, dz}{\sqrt{a^2 - x^2 - y^2 - z^2}}$ the integral being extended to all positive values of the variable for which the expression is real.

27) Evaluate $\int_0^{2\pi} \int_0^{\pi/4} \int_0^a r^2\, sin\theta\, dr\, d\theta\, d\emptyset$.

28) Find the area in the 1st co. ordinate included between the parabola $x^2 = 16y$ the y axis and line $y = 2$

29) Find the area enclosed by ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

30) Find the smaller of the area bounded by $y = 2 - x$; $x^2 + y^2 = 4$.

10 Marks :

31) Evaluate $\iint\limits_R (x^2 + y^2)\, dx\, dy$. Where R is the region in the positive quadrant for which $x + y \le 1$

32) Evaluate $\int_0^{\pi/2} \int_0^{\pi/2} sin(\theta + \emptyset)\, d\theta\, d\emptyset$.

33) Evaluate $\iint (x + y)^2\ dx\, dy$ over the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

34) Evaluate $\iint (x^2 + y^2)\ \ dx\, dy$ over the area bounded by the curve y= $4x, x + y = 3, y = 0, y = 2$.

35) Transform the integral into polar co-ordinates and hence evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2 + y^2}\ dy\, dx$.

36) Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} (x^2 + y^2)\ dy\, dx$ by changing into polar co-ordinates.

37) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{dz\, dy\, dx}{(x+y+z+1)^3}$

38) Evaluate $\iiint\limits_R (x - y + z) dx\, dy\, dz$ Where R is given by $1 \le x \le 2;\ 2 \le y \le 3;\ 1 \le z \le 3;$

39) Evaluate $\iint \frac{x^2y^2}{x^2+y^2}\ dx\, dy$ over the annuear region bounded the circles $x^2 + y^2 = a^2;\ x^2 + y^2 = b^2;\ x^2 + y^2 = b^2$ (b>a) by transforming into polar co ordinates.

40) Find the area included between the curves $y^2 = 4x;\ x^2 = 4y$.

UNIT - V

Choose the correct Answer :

1) $\int sec^2 x \cos x\, dx$ =?

a) $\frac{1}{2} cos^2 x + c$

b) $\frac{1}{3} cos^3 x + c$

c) $\frac{1}{2} sin^2 x + c$

d) $\frac{1}{3} sin^3 x + c$

2) What is fourier series?

a) The representation of periodic signals in a mathematical manner is called a fourier series.

b) The representation of non periodic signals in a mathematical manner is called a fourier series.

c) The representation of non periodic signals in terms of complex exponentials or sinusoids is called a fourier series.

d) The representation of periodic signals in terms of complex exponentials or sinusoids is called a fourier series.

3) Which of the following is an "even" function of t ?

a) t^2

b) $t^2 - 4t$

c) $\sin(2t) + 3t$

d) $t^3 + 6$

4) A "periodic function" is given by a function which

a) Has a period $T = 2\pi$

b) Satisfies $f(t+T) = f(t)$

c) Satisfies $f(t+T) = -f(t)$

d) Has a period $T = \pi$

5) For the given periodic function $f(t) = \begin{cases} 2t & for\ 0 \le 1 \le 2 \\ 4 & for 2\ \le t \le 6\ \ (= T) \end{cases}$ The coefficient b_1 of the continuous fourier series associated with the given function $f(t)$ can be computed as

a) -75.6800

b) -7.5680

c) -6.8968

d) -0.7468

6) For the given periodic function $f(t) = \begin{cases} 2t\ for\ 0 \le t \le 2 \\ 4\ for\ 2 \le t \le 6 \end{cases}$ with a period T=6.

The fourier coefficient a_1 can be computed as.

a) -9.2642

b) -8.1275

c) -0.9119

d) -0.5116

7) Who discovered Fourier series?

a) Jean Baptiste Fourier

b) Jean Baptiste Joseph Fourier

c) Fourier Joseph

d) Jean Fourier

8) In half range cosine Fourier series, we assume the function to be ………..

a) Odd function

b) Even function

c) Can't be determined

d) Can be anything

9) Find b_n when we have to find the half range sine series, we assume the function x^2 in the interval 0 to 3.

a) $-18\ \frac{\cos(n\pi)}{n\pi}$

b) $18\ \frac{\cos(n\pi)}{n\pi}$

c) $-18\ \frac{\cos(n\pi/2)}{n\pi}$

d) $18\ \frac{\cos(n\pi/2)}{n\pi}$

10) In parseval's relation of Half range fourier cosine series expansion, which of the following terms doesn't appear?

a) a_0

b) a_n

c) b_n

d) All terms appear

Answers :

1) d 2) d 3) a 4) b 5) d 6) c 7) b 8) b 9) a 10) c

2 Marks :

11) To solve $\int x\ sinnx\ dx$

12) To solve $\int x\ cosnx\ dx$

13) Define Even function with examples.

14) Define Odd function with examples.

15) Let $f(x)$ be a function of period 2π such that $f(x) = \begin{cases} 1, -\pi < x < 0 \\ 0, \ \ 0 < x < \pi \end{cases}$

16) Let $f(x)$ be a function of period 2π such that $f(x) = \begin{cases} 0, \ -\pi < x < 0 \\ x, \ \ 0 < x < \pi \end{cases}$

17) Let $f(x)$ be a function of period 2π such that $f(x) = \frac{x}{2}$ over the interval $0 < x < 2\pi$.

18) Let $f(x)$ be a function of period 2π such that $f(x) = \begin{cases} \pi - x, \ 0 < x < \pi \\ 0, \ \ \pi < x < 2\pi \end{cases}$

19) Let $f(x)$ be a function of period 2π such that $f(x) = x$ in the range $-\pi < x < \pi$.

20) To prove $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$

5 Marks :

21) To solve $\int_0^1 x^2 \ sinnx \ dx$.

22) To solve $\int \ x^2 \ cosnx \ dx$.

23) To prove $f(x) \ = \ a \ in \ 0$ to π

$= \ -a \ in \ \pi$ to 2π

24) Explain Half range – fourier series.

25) Find a sine series to for f(x)=c in the range 0 to π

26) Find a cosine series corresponding to the function $f(x) = x$ defind in the intravel 0 to π.

27) Find the half series cosine f$(x) = sinnx$ the range 0 to π.

28) Explain Change the interval.

29) The range $0 \ to \ 2l \ f(x)$ is defined by the relation. Relation $f(x) = \begin{cases} 0 \ \ where \ 0 < x < l \\ a \ where \ l < x < 2l \end{cases}$

30) If $f(x) = \begin{cases} 0 \quad when \ -l < x < 0 \\ 1 \quad when \ \ 0 < x < l \end{cases}$

10 marks :

31) If $f(x) = x$; 0 to 2π in Fourier series with period 2π.

32) Express$f(x) = x^2$; $0 \leq x \leq 2\pi$ in fourier series with period 2π.

33) Express $f(x) = x \ (-\pi < x < \pi)$ as a fourier series with period 2π.

34) Express $f(x) = x^2 \ (-\pi < x < \pi)$ as a fourier series with period 2π.

35) Express $f(x) = \pi^2 - x^2$ as a fourier series in the intravel $-\pi$ to π.

36) A function $f(x)$ we find within range 0, 2π by the relation.

37) $f(x) = |x|. -\pi < x < \pi$ and hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{8}$

38) Show that $x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n cosnx}{n^2}$ in the interval $(-\pi < x < \pi)$Deduce that,

i) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots = \frac{\pi^2}{12}$

ii) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$

iii) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$

39) Express as a fourier series $f(x) = x$ in $(-\pi < x < \pi)$.

40) To prove $e^x = \frac{sinh\pi x}{\pi}\left[1 + 2\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1}\ (cosnx - sinnx)\right]$

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