ஸ்ரீ-ல-ஸ்ரீ காசிவாசி சுவாமிநாத சுவாமிகள் கலைக் கல்லூரி

திருப்பனந்தாள் – 612504

S.K.S.S ARTS COLLEGE, THIRUPPANANDAL - 612504

QUESTION BANK

Title of the Paper

REAL ANALYSIS

COURSE – III MATHS

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CORE COURSE X

REAL ANALYSIS

Objectives: To enable the students to

1. Understand the real number system and countable concepts in real number system
2. Provide a Comprehensive idea about the real number system.
3. Understand the concepts of Continuity, Differentiation and Riemann Integrals
4. Learn Rolle's Theorem and apply the Rolle's theorem concepts.

UNIT I :

Real Number system – Field axioms –Order relation in R. Absolute value of a real number & its properties –Supremum & Infimum of a set – Order completeness property – Countable & uncountable sets.

UNIT II :

Continuous functions –Limit of a Function – Algebra of Limits – Continuity of a function –Types of discontinuities – Elementary properties of continuous functions – Uniform continuity of a function.

UNIT III :

Differentiability of a function –Derivability & Continuity –Algebra of derivatives – Inverse Function Theorem – Daurboux"s Theorem on derivatives.

UNIT IV :

Rolle's Theorem –Mean Value Theorems on derivatives- Taylor's Theorem with remainder- Power series expansion .

UNIT V :

Riemann integration –definition – Daurboux's theorem –conditions for integrability – Integrability of continuous & monotonic functions - Integral functions – Properties of Integrable functions - Continuity & derivability of integral functions – The Fundamental Theorem of Calculus and the First Mean Value Theorem.

TEXT BOOK(S) :

1. M.K,Singhal & Asha Rani Singhal , A First Course in Real Analysis, R.Chand & Co., June 1997 Edition
2. Shanthi Narayan, A Course of Mathematical Analysis, S. Chand & Co., 1995

UNIT – I - Chapter 1 of [1]
UNIT – II - Chapter 5 of [1]
UNIT – III - Chapter 6 – Sec 1 to 5 of [1]
UNIT – IV - Chapter 8 – Sec 1 to 6 of [1]
UNIT – V - Chapter 6 – Sec 6.2, 6.3, 6.5, 6.7, 6.9 of [2]

REFERENCE(S) :

1. Goldberge, Richard R, Methods of Real Analysis, Oxford & IBHP Publishing Co., New Delhi, 1970.

UNIT - I

Choose the correct Answer :

1) The set R is close with respect to addition. That is, if a and b any two real numbers, then $a + b$ is a unique real number.

   a) Closure for addition.

   b) Associative law of addition.

   c) Identity element for addition.

   d) Commutative law of addition.

2) The operation of addition in R is associative. That is, for each triple of real numbers a,b,c, $a + (b + c) = (a + b) + c$.

   a) Existence of negatives.

   b) Closure for addition.

   c) Associative law of addition.

   d) None of these.

3) There exists a real number, namely 1, such that $a.1 = 1.a = a$ for all $a \in R$

   a) Associative law of multiplication.

   b) Identity element for multiplication.

   c) Existence of inverses.

   d) None of these.

4) For each pair of real numbers a and b, $ab = ba$

   a) Identity element for multiplication.

   b) Existence of inverses.

   c) Associative law of multiplication.

   d) Commutative law of multiplication.

5) The difference between to real numbers x and y is given by x+(-y), and is denoted by x-y. The operation of finding the difference is called ……….

   a) Addition.

   b) Multiplication.

   c) Subtraction.

   d) Division.

6) A real number a is said to be ……. If a>0.

   a) Positive.

b) Negative.

c) Equal.

d) Infinity.

7) If x be a real number, then its ……….. denoted by $|x|$, is defined by the rule

$$|x| = \begin{cases} x & if\ x \geq 0, \\ -x & if\ x < 0, \end{cases}$$

We may observe that $|x|$ is defined for every $x \in R$. Also, $x_1 = x_2 \Rightarrow |x_1| = |x_2|$.

a) Less then

b) Absolute value

c) Greater then

d) None of the above

8) If for a set S of real numbers, there exists a real number u, such that $x \in S \Rightarrow x \leq u$, then u is called an ………… of S.

a) Upper bound

b) Lower bound

c) Both a and b

d) None of the above.

9) If the set of all upper bound of a set S of real numbers has a smallest member, say w, then w is said to be a ……. of S.

a) Bounded

b) Infimum

c) Supremum

d) None of these

10) A real number is said to be …….. if it is the root of some polynomial equation with rational coefficients.

a) Finite

b) Infinite

c) Enumerable

d) Algebraic

Answers :

1) a 2) c 3) b 4) d 5) c 6) a 7) b 8) a 9) c 10) d

2 Marks :

11) Define Real number.

12) Define Commutative law of multiplication.

13) If $x, y, z \epsilon R$ and $x + y = y + z$ then $x = y$.

14) Definition of Quotient.

15) For ever $x \epsilon R$ to prove $|x| = |-x|$.

16) Define Upper bound.

17) Define Supermum.

18) Define Infimum (or) Greatest lower bound.

19) Define Finite set.

20) Define Algebraic number.

5 Marks :

21) For each $x \epsilon R$ To find

i) $-(-x) = x$

ii) $-(x + y) = (-x) + (-y)$

22) There can exist at the most one identity for addition in R (uniqueness of zero)

23) To each x in R there correspond one and only one real number y such that

$x + y = y + x = 0$. (Uniqueness of negative)

24) There can exist at the most one identity element for multiplication in R.

25) If x, y be the real number such that $xy = 0$ then either $x = 0$ (or) $y = 0$.

26) If x, y, z be the real numbers such that $xz = yz$ and $z \neq 0$ then $x = y$.

27) If a,b be positive real numbers then ab is a positive real number.

28) For every $x \epsilon R$ $|x|^2 = x^2 = |-x|^2$

29) The set of positive real number R^+ is not bounded above.

30) A set cannot have more than one supremum.

10 Marks :

31) State and prove Triangle inequality.

32) For all real number x and y $|x - y| \geq \big||x| - |y|\big|$

33) To prove $|x + y|^2 + |x - y|^2 = 2(|x|^2 + |y|^2)$.

34) To prove $|x - y| \leq |x| + |y|$

35) Any non empty set of real numbers which is bounded below infimum.

36) If x and y ne any positive real numbers, then there exists a positive integer n such that $ny > x$ (lower bound)

37) There is no rational number whose square is 2.

38) The set of rational numbers is not complete ordered field.

39) Every subset of a countable set is countable.

40) The set $[0,1]$ is uncountable.

UNIT - II

Choose the correct Answer :

1) A function f defined on an open interval I is said to be continuous from the……. at $x_0 \in I$ if $\lim_{x \to x_0 - 0} f(x)$ exists and equal $f(x_0)$.
   a) Right
   b) Left
   c) Center
   d) Equal

2) If $\lim_{x \to c+0} f(x) = l$ or $\lim_{x \to c} f(x) = l$ this is ………
   a) Right hand limits

b) Left hand limits

c) Limits

d) None of these

3) Let c be any real number and let f be defined on R by setting $f(x) = c$ for all $x \in R$.

a) Identity function

b) zero value

c) Constant

d) None of these

4) Let f be a function defined on R by setting $f(x) = x$ for all $x \in R$

a) Constant function

b) nonconstant function

c) Identity function

d) None of the above

5) Let f be defined on R by setting

f(x)=1, when x is rational.

f(x) = -1, when x is irrational.

a) Identity function

b) Constant function

c) nonconstant function

d) Dirichlet's function

6) f has a ……… of P. if $\lim_{x \to p} f(x)$ exists but it is not equal to f(p).

a) Continuity

b) Discontinuity

c) Removable discontinuity

d) Both a and b.

7) Let f and g defined on interval I. If f and g are continuous at $p \in I$. Then $f + g$ is …… at p.

a) Continuous

b) Discontinuous

c) Both a and b

d) None of these

8) A function f is defined on an interval I is said to be ……… on I, if given $\in > 0$, there exists a $\delta > 0$ such that if $x, y \in I$ and $|x - y| < \delta$ then $|f(x) - f(y)| < \in$.

a) Uniformly continuous

b) Discontinuity

c) Removable discontinuity

d) Both b and c

9) If $\lim_{x \to c} f(x) = l$, $\lim_{x \to c} g(x) = m$ then $\lim_{x \to c} \left(\frac{f}{g}\right)(x) =$ …….. Provided $m \neq 0$

a) $l - m$

b) l/m

c) lm

$d)$ $l + m$

10) If f be a continuous one to one function on the closed interval [a,b] then f^{-1} is ………

a) Dirichlet's function

b) Discontinuous

c) Identity function

d) None of these

Answers :

1) b 2) a 3) c 4) c 5) d 6) c 7) a 8) a 9) b 10) d

2 Marks :

11) Define Limits.

12) If $\lim_{x \to c} f(x) = l, \lim_{x \to c} g(x) = m$ then $\lim_{x \to c} (f/g)(x) = l/m$ provided $m \neq 0$.

13) Define continuous function.

14) Prove that constant function is continuous.

15) Prove that identity function is continuous.

16) Define Dirichlet's function.

17) Define Type of Discontinuity.

18) Define Removable discontinuity.

19) Statement of intermediate value theorem.

20) Define uniform continuity.

5 Marks :

21) Let f and g be defined on some neighborhood of c. If $\lim_{x\to c} f(x) = l$ and $\lim_{x\to c} g(x) = m$ then $\lim_{x\to c} (f+g)(x) = l+m$.
22) If $\lim_{x\to c} f(x) = l$ then $\lim_{x\to c} |f(x)| = |l|$.
23) The function f defined on R by $f(x) = x^2$ for all $x\epsilon R$ is continuous.
24) Let f be the function defined on R by

$$f(x) = \begin{cases} \sin\frac{x}{x}, & if\ x \neq 0 \\ 0, & if\ x = 0 \end{cases}$$

(Discuss or prove that) F has a removable discontinuity at $x = 0$ (nature of discontinuity or continuity of f at $x = 0$)

25) Let f and g defined on interval I. If f and g are continuous at $P \epsilon I$, then $f+g$ is continuous at p.
26) Let f and g defined on interval I. If f and g are continuous at $P \epsilon I$, then fg is continuous at p.
27) If f is continuous at a point p and ϵR , then cf is continuous at p.
28) Let f and g be defined on an interval I and let $g(p) \neq 0$. If f and g are continuous at $P \epsilon I$, then f/g is continuous.
29) Let f and g be defined on an interval I. If they are both continuous at $P \epsilon I$. Then the functions $max\{f,g\}$ and $min\{f,g\}$ are both continuous at p.
30) Let f is continuous then $|f|$ is continuous.

10 Marks :

31) If $\lim_{x\to c} f(x) = l$ and $\lim_{x\to c} f(x) = m$ then $= m$.
32) Let f and g be defined on some neighborhood of c. If $\lim_{x\to c} f(x) = l$ and $\lim_{x\to c} g(x) = m$ then $\lim_{x\to c}(fg)(x) = lm$.
33) If $\lim_{x\to c} g(x) = m$ and $m \neq 0$ then $\lim_{x\to c} \frac{1}{g(x)} = \frac{1}{m}$
34) A function f defined on $I \subset R$ is continuous at $P \in I$ iff for every sequence $< pn >$ in I which converges to P. We have $\lim_{n\to\infty} f(pn) = f(p)$.
35) A function f defined on R is continuous on R iff for each set G in R $f^{-1}(G)$ is an open set in R.

36) A function f defined on R is continuous on R iff for each closed set F in R. $f^{-1}(F)$ is also a closed set in R.

37) Let f and g be defined on an interval I and J respectively and let $f(I) \subset f(J)$. If f is continuous at $P \in I$ and g is continuous at $f(p)$ then (gof) is continuous at p.

38) If f be a continuous on closed interval [a,b] and f(a)<0<f(b) then there exists a point x belongs to (a,b) such that f(x)=0.

39) State and prove intermediate value theorem.

40) If f be a continuous one to one function onto the closed interval [a,b] then f^{-1} is also continuous.

UNIT - III

Choose the correct Answer :

1) If a function f is derivable at a point x_0, then for each real number c, the function cf is also derivable at x_0, and ………
   a) $(cf)'(x_0) = cf'(x_0)$
   b) $(cf)(x_0) = cf(x_0)$
   c) $(f)'(x_0) = f'(x_0)$
   d) $(cf)''(x_0) = cf''(x_0)$

2) Differentiate of the $tan^{-1}x^2$
   a) $\frac{2}{1+x^4}$
   b) $\frac{4x}{1+x^4}$
   c) $\frac{2x}{1+x^4}$
   d) $\frac{1}{1+x^4}$

3) Differentiate of the $sinh^2x$
   a) $2\ sinx\ cosx$
   b) $2\ sinhx\ coshx$
   c) $sinhx\ coshx$
   d) $sinx\ cosx$

4) Differentiate of the $sinx\ coshx$

a) $Coshx\ coshx + sinhx\ sinhx$

b) $Cosx\ + Sinx$

c) $Coshx + Sinhx$

d) $Cosx\ coshx + sinx\ sinhx$

5) Find $\frac{dy}{dx}$ for each of the implicitly defined function $ax^2 + 2hxy + by^2 = 1.$

a) $\frac{ax+hy}{hx+by}$

b) $-\frac{ax+hy}{hx+by}$

c) $-\frac{ay+hx}{hy+bx}$

d) $\frac{ay+hx}{hy+bx}$

6) The function f is continuous at $a \in M$ if $\lim_{x\to a} f(x)$ = ……..

a) $f(b)$

b) $f(c)$

c) $f(x)$

d) $f(a)$

7) Find $\frac{dy}{dx}$ for each of the implicitly defined function $x = y\ ln\ (xy)$

a) $\frac{y(x+y)}{x(x+y)}$

b) $\frac{y(x-y)}{x(x-y)}$

c) $\frac{y(x-y)}{x(x+y)}$

d) $\frac{y(x+y)}{x(x-y)}$

8) If real value function f as continuous at the point $a \in R$ if given $\epsilon > 0$ there exist $\delta > 0$ such that ……….

a) 0

b) 1

c) $|f(x) - f(a)| < \epsilon$

d) ∅

9) Find $\frac{dy}{dx}$ for each of the implicitly defined function $x^m y^n = (x + y)^{m+n}$.

a) $\frac{y}{x}$

b) $\frac{x}{y}$

c) *1*

d) 0

10) Find $\frac{dy}{dx}$ in $= a\cos\theta, y = a\sin\theta.$

a) cotθ

b) cosθ

c) −cotθ

d) −tanθ

Answers :

1) a 2) c 3) b 4) d 5) b 6) d 7) c 8) c 9) a 10) c

2 Marks :

11) Define Derivability of an open interval.
12) Prove that identity function $f(x) = x \; \forall \; x \in R$ is derivable in R .
13) Prove that constant function $f(x) = c \; \forall \; x \in R$ is derivable on R.
14) Let f be a function defined on R by $f(x) = x^n \; \forall \; x \in R$. Prove that f is derivable on R.
15) f(x) = |x| check whether f is derivable at x=0.
16) Statement of Chain Rule.
17) Statement of Darbouxs theorem.
18) Let f be defined on R by setting

f(x)=|x − 1|, for all $x \in R$

show that f is derivable at all points except x=1. Also show that $Rf^1(1) = 1, Lf^1(1) = -1$

19) Show that the function f defined on R by setting

$f(x) = x \sin(1/x), if \; x \neq 0.$

$f(0) = 0.$

is not derivable at $x = 0$.

20) Let f be defined on R by setting

$f(x) = |x - 2| + |x + 2|$, for all $x \in R.$

show that f is not derivable at the points $x = -2$ and $x = 2$, and is derivable at every other point.

5 Marks :

21) Let f be a function defined on an interval I. If f be derivable at a point x_o belongs to I, Then it is continuous at x_o (Derivability ⇒ Continuity).

22) If a function f is derivable at x_o, then the function cf is also derivable at x_o, for each real number c is also $(cf)^1 x_o = cf^1(x_o)$.

23) Let f and g be two function defined on I. If f and g are derivable at $x_o \in I$, then also if $(f + g)$ *and* $(f + g)^1 x_o = f^1(x_o) + g^1(x_o)$.

24) Let f be defined on R by setting

f(x) = $\begin{cases} 0, if\ x \leq 0. \\ x, if\ x > 0. \end{cases}$

Show that f is not derivable at x=0 and is derivable at every other point.

25) Let f be the function defined on R by setting.

$f(x) = x^3 sin\,(1/x), if\ x \neq 0.$

$f(0) = 0.$

Show that f^1 is continuous on R but is not derivable at x=0.

26) Let f be the function defined on R by setting.

$f(x) = x^4 sin\,(1/x), if\ x \neq 0.$

$f(0) = 0.$

27) Let f be defined on R by setting

$f(x) = \frac{e^{-1/x} - e^{1/x}}{e^{-1/x} + e^{1/x}}$, if $x \neq 0$

$f(0) = 0$, if $x = 0$

show that f is not derivable at x=0. Do $Rf^1(1)$ and $Lf^1(1)$ exist?

28) Let f be defined on R by setting

$f(x) = x^{2n}\ sin\left(\frac{1}{x}\right)$ *if* $x \neq 0$, and $f(0) = 0.$

prove that f^n exists for all $x \in R$ but f^n is not continuous at x=0.

29) Let f be defined on R by setting

$f(x) = x^{2n+1}\ sin\left(\frac{1}{x}\right)$ if $x \neq 0$, and $f(0) = 0.$

prove that f^n exists for all $x \in R$, f^n is continuous at x=0, but f^n is not derivable at x=0.

30) Let f be a function with domain D and let g be the function defined on D by setting $g(x) = xf(x)$, for all $x \in D$. Prove that if f be continuous at x=0, then g is derivable at x=0.

10 Marks :

31) If f and g be two functions defined on I. If f and g are derivable at $x_0 \in I$. Then also if fg and also $(fg)^1 x_0 = f^1(x_0).g(x_0) + f(x_0).g^1(x_0)$.
32) Let f be derivable at x_0 and let $f(x_0) \neq 0$.Then the function $1/f$ is derivable at x_0 and also $(1/f)^1 x_0 = -f^1(x_0)/f(x_0)^2$.
33) State and prove Chain Rule theorem.
34) Let f be a continuous one to one function defined on an interval and let f be derivable at x_0, with $f^1(x_0) \neq 0$. Then the inverse of the function f is derivable at $f(x_0)$ and its derivative at $f(x_0)$ is $1/ f(x_0)$.
35) State and prove Darboux's theorem.
36) If f and g be two functions having the same domain D, and if $f + g$ be derivable at $x_0 \in D$, is it necessary that f and g be both derivable at x_0.
37) If f and g be tow functions having the same domain D, and if fg be derivable at $x_0 \in D$, is it necessary that f and g be both derivable at x_0?
38) If f be derivable at x_0, then show that $|f|$ is also derivable at x_0, provided $f(x_0) \neq 0$.
39) Show by means of an example that if $f(x_0) = 0$, then f may be derivable at x_0 and $|f|$ may not be derivable at x_0.
40) If f is defined and derivable on$[a, b]$, $f(a) = f(b) = 0$, and $f^1(a)$ and $f^1(b)$ are of the same sign, then prove that f must vanish at least once in $]a, b[$.

UNIT - IV

Choose the correct Answer :

1) Find the number θ that appears in the conclusion of Lagrange's mean value theorem in $f(x) = x^2; a = 1, h = 1/5$.

   a) $\sqrt{\left(\frac{1}{3}\right) - 5}$

b) $\sqrt{\left(\frac{91}{3}\right) - 5}$

c) $\sqrt{\left(\frac{9}{3}\right) - 5}$

d) $\sqrt{\left(\frac{1}{93}\right) - 5}$

2) Find the number θ that appears in the conclusion of Lagrange's mean value theorem in $f(x) = log_e x; a = 1, h = 1$

a) $(log_e 1.1)^{-1} - 10$

b) $(log_e 1.1)^{-1} - 1$

c) $(log_e 1.10)^{-1} - 1$

d) $(log_e 1.10)^{-1} - 10$

3) Calculate a value of c for which

$$\frac{f(c) - f(a)}{g(b) - g(c)} = \frac{f^1(c)}{g^1(c)}$$

for each of the following pairs of function.

$f(x) = x^2, g(x) = x^4;\ a = 2, b = 4$

a) $\sqrt{20}$

b $\sqrt{30}$

c) $\sqrt{10}$

d) $\sqrt{15}$

4) The value of c in Rolle's theorem for the function $f(x) = x^3 - 3x$ in the interval $[0, \sqrt{3}]$ is

a) 1

b) -1

c) $3/2$

d) $1/3$

5) Suppose $f: [a, b] \rightarrow R$ is continuous on $[a, b]$ and f is differentiable on (a, b). If $f(a) = f(b) = 0$, there is c$\in$ (a, b): $f'(c) = 0$ ……….

a) Rolle's theorem

b) Mean value theorem

c) Intermediate value theorem

d) None of these

6) For the function $f(x) = x^2 - 2x + 1$. We have Rolles point at x=1. The coordinate axes are then rotated by 45 degrees in anticlockwise sense. What is the position of new Rolles point with respect to the transformed coordinate axes?

a) 1/2

b) 5/2

c) 3/2

d) 1

7) The necessary condition for the maclaurin expansion to be true for function $f(x)$ is ……….

a) f(x) should be continuous

b) f(x) should be differentiable

c) f(x) should exists at every point

d) f(x) should be continuous and differentiable

8) The expansion of $e^{sin(x)}$ is?

a) $1 + x + \frac{x^2}{2} + \frac{x^4}{8} + \cdots$

b) $1 + x + \frac{x^2}{2} - \frac{x^4}{8} + \cdots$

c) $1 + x - \frac{x^2}{2} + \frac{x^4}{8} + \cdots$

d) $1 + x + \frac{x^3}{6} - \frac{x^5}{10} + \cdots$

9) Given $f(x) = In(cos(x))$,calculate the value of $In\left(cos\left(\frac{\pi}{2}\right)\right)$.

a) -1.741

b) 1.741

c) 1.563

d) -1.563

10) The explicit formula for the geometric sequence 3,15,75,375,… is ……….

a) $2 * 6! * 3^{n-1}$

b) $3 * 5^{n-1}$

c) $3! * 8^{n-1}$

d) $7 * 4^{n-1}$

Answers :

1) b 2) a 3) c 4) a 5) a 6) c 7) d 8) b 9) a 10) b

2 Marks :

11) Find 'c' of Lagrange's mean value theorem of $f(x) = x^2$ where $a = 1, h = 1/5$.
12) Define Geometric significance of Rolle's theorem.
13) Statement of Rolle's theorem.
14) Verify the hypotheses and the conclusion of Rolle's theorem for the function f defined on [a,b] in each of the following case $f(x) = x^2, a = -1, b = 1$.
15) Verify the hypotheses and the conclusion of Rolle's theorem for the function f defined on [a,b] in each of the following case $f(x) = cos\, x\, , a = -\pi/2,\ b = \pi/2$.
16) Statement of Lagrange's mean value theorem.
17) Statement of Cauchy's mean value theorem.
18) Statement of generalized mean value theorem.
19) $f(x) = x^2 - 2x + 3$, where $a = 1, h = 1/2$.
20) Write the power series expansion of some standard functions.

5 Marks :

21) If f is defined and continuous on [a,b] and is derivable in (a,b) and if $f^1(x) = 0\ \forall\ x \in (a,b)$ then f(x) has a constant value throughout the interval [a,b]
22) State and prove Generalized mean value theorem.
23) Find the Maclaurin's of the function $f(x) = e^x$.
24) Find the Maclaurin's expansion for the function $f(x) = cosx$.
25) Prove that if p be any polynomial and p^1 the derivative of p, then between any two consecutive zeros of p^1, there lies at the most one zero of p.
26) Show that there is no real number k for which the equation $x^3 - 3x + k = 0$, has two distinct roots in [0,1].
27) Verify that on the curve $y = px^2 + qx + r(p, q, r$ being real numbers,p$\neq 0$), the chord joining the points for which $x = a, x = b$ is parallel to the tangent at the point given by $x = 1/2(a + b)$.

28) Prove that for any quadratic function $px^2 + qx + r$, the value of θ in Lagrange's theorem is always 1/2, whatever p,q,r,a,h may be.

29) Calculate a value of c for which

$$\frac{f(c) - f(a)}{g(b) - g(c)} = \frac{f^1(c)}{g^1(c)}$$

for each of the following pairs of function.

$f(x) = e^x, g(x) = e^{-x}; a = 0, b = 1$

30) Calculate a value of c for which

$$\frac{f(c) - f(a)}{g(b) - g(c)} = \frac{f^1(c)}{g^1(c)}$$

for each of the following pairs of function.

$f(x) = sinx, g(x) = cosx;\ a = -\pi/2, b = 0$

10 Marks :

31) State and prove Rolle's therorem.

32) State and prove Lagrange's mean value theorem.

33) Find c of Lagrange's mean value theorem if $f(x) = x(x-1)(x-2)$ where $a = 0,\ b = 1/2.$

34) State and prove Cauchy's mean value theorem.

35) State and prove Taylor's theorem with Lagr4ange's form of reminder.

36) State and prove Taylor's theorem with Cauchy's from of remainder.

37) Find the Maclaurines expansion for the function of $f(x) = sinx$

38) Find the Maclaurine's expansion for the function $f(x) = (1 + x)^m$.

39) Calculate a value of c for which

$$\frac{f(c) - f(a)}{g(b) - g(c)} = \frac{f'(c)}{g'(c)}$$

for each of the following pairs of function.

$f(x) = x^2, g(x) = x; a = 0, b = 1$

40) If $f''(x)$ be defined on [a,b] and if $|f''(x)| \leq M$, for all x in [a,b], and if c be any real number between a and b, then prove that

$$\left| f(c) - f(a) - (c-a)\left\{\frac{f(b) - f(a)}{b - a}\right\} \right| \leq \frac{1}{8}(b-a)^2 M.$$

UNIT - V

Choose the correct Answer :

1) If f is a Riemann integrable function on [a,b] and λ is any real number then following holds ……

   a) $\lambda \int_a^b f(x)dx = \int_a^b \lambda f(x)dx$

   b) $\lambda \int_a^b f(x)dx \neq \int_a^b \lambda f(x)dx$

   c) $\lambda \int_a^b f(x)dx \geq \int_a^b \lambda f(x)dx$

   d) None of these

2) If $\int_{-a}^{b} f\, dx$ and $\int_{a}^{-b} f\, dx$ are lower and upper Riemann integrable on [a,b]. Then …….

   a) $\int_{-a}^{b} f\, dx \geq \int_{a}^{-b} f\, dx$

   b) $\int_{-a}^{b} f\, dx = \int_{a}^{-b} f\, dx$

   c) $\int_{-a}^{b} f\, dx \leq \int_{a}^{-b} f\, dx$

   d) $\int_{-a}^{b} f\, dx \neq \int_{a}^{-b} f\, dx$

3) $f: [a, b] \to R, f$ is Riemann integrable then …….

   a) $\left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|dx$

   b) $\int_a^b |f(x)|dx \leq \left|\int_a^b f(x)dx\right|$

   c) $\int_a^b |f(x)|dx = \left|\int_a^b f(x)dx\right|$

   d) $\left|\int_a^b f(x)dx\right| = \int_a^b |f(x)|dx$

4) If f is Riemann integrable with respect to α on [a,b], then ………

   a) f is increasing and α is bounded function

   b) f is bounded and α is increasing function

   c) f and α are both bounded

   d) f and α are both increasing

5) A function is a monotonic function f if ………

   a) f is only a decreasing function

   b) f is only a increasing function

   c) f is either increasing or decreasing function

d) None of these

6) Singular monotonic function on [a,b] have ………

a) $f(x) = 0, \forall\, x \in [a,b]$

b) $f'(x) = 0, \forall\, x \in [a,b]$

c) $f'(x) \neq 0, \forall\, x \in [a,b]$

d) $f(x) \neq 0, \forall\, x \in [a,b]$

7) If f is bounded variation on [a,b] iff …….

a) f is the quotient of two monotone real valued function on [a,b]

b) f is the product of two monotone real valued function on [a,b]

c) f is the difference of two monotone real valued function on [a,b]

d) None of these

8) If f is integrable on [a,b], then the function $F(x) = \int_a^x f'(t)dt$ ……..

a) A continuous function of bounded variation on [a,b]

b) A discontinuous function of bounded variation on [a,b]

c) A discrete function of bounded variation on [a,b]

d) None of this

9) If f is integrable on [a,b] and $\int_a^x f(t)dt = 0$ for all $x \in |a,b|$, then ……….

a) $f(t) = 0$ nowhere in [a,b]

b) $f(t) = 0$ almost everywhere in [a,b]

c) $f(t)$ is not equal to zero almost everywhere in [a,b]

d) None of these

10) If ∅ is a convex function on $(-\infty, \infty)$ and f an intergrable function on [0,1] then

a) $\int \emptyset \mathrm{f(t)dt} \geq \emptyset[\int \mathrm{f(t)dt}]$

b) $\int \emptyset f(t)dt = \emptyset[\int f(t)dt]$

c) $\int \emptyset f(t)dt \leq \emptyset[\int f(t)dt]$

d) $\int \emptyset f(t)dt < \emptyset[\int f(t)dt]$

Answers :

1) a 2) c 3) b 4) b 5) c 6) b 7) c 8) a 9) b 10) a

2 Marks :

11) Define Refinement of a partition.

12) Define partition of a closed interval.

13) Define Norm of a partition.

14) Define Upper integral.

15) Define Lower integral.

16) Define Riemann integral.

17) Statement of conditions for integrability.

18) Statement of second form on integrability.

19) Statement of integrability of product.

20) Statement of integrability Quotient.

5 Marks :

21) Let I=[a,b] be a finite closed interval of $a = x_0 < x_1 < x_2 < \ldots < x_n = b$ then the finite order set $D = \{x_0, x_1, \ldots x_n\}$ is called a partition of I.

22) Explain Upper and lower darboux sums.

23) Explain oscillatory sum.

24) If f is monotonic in [a,b] then it integrable in [a,b].

25) Equivalence of the two definition of the Riemann integral.

26) Every continuous function is integrable.

27) State and prove continuity of the integral function.

28) State and prove Derivability of the integral function..

29) Prove if $f'(x) > 0$ in]a,∞[and f is continuous at a, then f is strictly increasing in [a, ∞[.

30) State and prove integrability of product.

10 marks :

31) $D_1 < D_2 \Rightarrow$ i) $s(D_1) \geq s(D_2)$.

ii) $S(D_1) \leq S(D_2)$.

ie) In proceeding from any divisions to a finer division the upper sum cannot increasing and the lower sum cannot decrease.

32) $\int_{a}^{\underline{b}} f(x)dx \geq \int_{\bar{a}}^{b} f(x)dx$ Prove that the lower Riemann integral cannot be greater than the upper Riemann integral.

33) To every $\epsilon > 0$ there corresponds a $\delta > 0$ such that $s(D) < \int_{a}^{\bar{b}} f(x)dx + \epsilon$ for every D such that $\|D\| \leq \delta$.

34) To every $\in > 0$ there corresponds a $\delta > 0$ such that S(D) < $\underline{\int_a^b} f(x)dx - \in$ for every division D such that $\|D\| < \delta$.

35) State and prove first form of condition for integrability.

36) State and prove integrability by parts.

37) State and prove First mean value theorem of integral calculus.

38) State and prove Second form of intergrability.

39) State and prove Fundamental theorem of integral calculus.

40) State and prove Second mean value theorem.

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