ஸ்ரீ-ல-ஸ்ரீ காசிவாசி சுவாமிநாத சுவாமிகள் கலைக் கல்லூரி

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QUESTION BANK

Title of the Paper

SEQUENCES AND SERIES

COURSE – II MATHS

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CORE COURSE V

SEQUENCES AND SERIES

OBJECTIVES :

1. To lay a good foundation for classical analysis
2. To study the behaviour of sequences and series.

Unit I :

Sequences – Bounded Sequences – Monotonic Sequences – Convergent Sequence – Divergent Sequences – Oscillating sequences

Unit II :

Algebra of Limits – Behaviour of Monotonic functions

Unit III

Some theorems on limits – sub sequences – limit points: Cauchy sequences

Unit IV :

Series – infinite series – Cauchy's general principal of convergence – Comparison – test theorem and test of convergence using comparison test (comparison test statement only, no proof)

Unit V :

Test of convergence using D Alembert's ratio test – Cauchy's root test – Alternating Series – Absolute Convergence (Statement only for all tests)

Book for Study :

Dr. S.Arumugam & Mr.A.Thangapandi Isaac Sequences and Series – New Gamma Publishing House – 2002 Edition.

Unit I : Chapter 3 : Sec. 3.0 – 3.5 Page No : 39-55

Unit II : Chapter 3 : Sec. 3.6, 3.7 Page No:56 – 82

Unit III : Chapter 3 : Sec. 3.8-3.11, Page No:82-102

Unit IV : Chapter 4 : Sec. (4.1 & 4.2) Page No : 112-128.

Unit V : Relevant part of Chapter 4 and Chapter 5: Sec. 5.1 & 5.2

Page No:157-167.

Book for Reference :

1. Algebra – Prof. S.Surya Narayan Iyer
2. Algebra – Prof. M.I.Francis Raj

UNIT - I

Choose the correct Answer :

1) The nth term of the sequence $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \ldots\right\}$ is ………

   a) $\frac{1}{n-1}$

   b) $\frac{1}{2n}$

   c) $\frac{1}{2(n+1)}$

   d) None of these

2) The following sequence $\{2,3,5,7,\ldots\}$ is a sequence of ……….

   a) Real number

   b) Prime number

   c) Even number

   d) Odd number

3) The nth Term of the sequence $\left\{1, \frac{1}{2}, 1, \frac{1}{3}, 1, \frac{1}{4}, \ldots\right\}$ is ………..

   a) For n even $\frac{1}{n-\frac{n}{2}}$ for n odd1

   b) For n even $\frac{1}{n+\frac{n}{2}+1}$ for n odd 1

   c) For n even $\frac{1}{n-\frac{n}{2}+1}$ for n odd 1

   d) For n even $\frac{1}{n+\frac{n}{2}}$ for n odd 1

4) The nth term of the sequence $\left\{2, \frac{-3}{2}, \frac{4}{3}, \frac{-5}{4}, \ldots\right\}$ is ……….

   a) $1+\frac{1}{n}$

b) $(-1)^{n-1}\left(1-\frac{1}{n}\right)$

c) $(-1)^{n-1}\left(1+\frac{1}{n}\right)$

d) None of these

5) A monotone sequence $\{a_n\}_{n=1}^{\infty}$ is convergent ……….

a) It is bounded

b) It is unbounded

c) It is decreasing

d) None of these

6) The sequence $\left[\frac{\cos\frac{n\pi}{2}}{n}\right]_{n=1}^{\infty}$ is …………

a) Convergent to 0

b) Divergent

c) Convergent to 1

d) None of these

7) A: Every bounded sequence is convergent

B: Every convergent sequence is bounded ………….

a) A and B are true.

b) A is true, B is false.

c) B is true, A is false.

d) A and B are false.

8) The sequence $\{1,0,1,0,1,0,\ldots\}$ is …………

a) Increasing sequence.

b) Decreasing sequence.

c) Monotone sequence.

d) None of these.

9) If $\{a_n\}_{n=0}^{\infty}$ converges to a, for all n, $a \geq 0$, then $\left\{\sqrt{a_n}\right\}_{n=0}^{\infty}$is ....

a) Converges to $\sqrt{a}$.

b) Diverges to $\sqrt{a}$

c) Converges to a

d) Diverges to a

10) If $\{a_n\}_{n=0}^{\infty}$ converges to A, then ....

a) $\{|a_n|\}_{n=0}^{\infty}$ Converges to A.

b) $\{|a_n|\}_{n=0}^{\infty}$ Converges to $|A|$.

c) $\{|a_n|\}_{n=0}^{\infty}$ is divergent sequence.

d) None of these.

Answers :

1) b 2) b 3) c 4) c 5) a 6) a 7) c 8) d 9) a 10) b

2 Marks :

11) Define Sequences.

12) Define Constant sequence.

13) Define Geometric sequence.

14) Definition Fibonacci sequence.

15) Define Bounded sequence.

16) Define Monotonic.

17) Define Convergent sequence.

18) Define Divergent sequence.

19) To prove $\lim_{n\to\infty} \frac{1}{n} = 0$

20) To prove $(n) \to \infty$.

5 Marks :

21) Write the first five terms of each of the following sequences$\left(\frac{(-1)^n}{n}\right)$.

22) Write the first five terms of each of the following sequences$\left(\frac{2}{3}\left(1 - \frac{1}{10^n}\right)\right)$.

23) Determine the range of the following sequences (n).

24) Determine the range of the following sequences (2n).

25) Determine which of the sequences given in examples of

i) Bounded above.

ii) Bounded below.

iii) Bounded.

26) Determine the l.u.b and g.l.b of the following sequences if 2,-2,1,-1,1,-1,…….

27) Show that if (a_n) is a monotonic sequence then $\left(\frac{a_1+a_2+\cdots a_n}{n}\right)$ is also monotonic sequence.

28) Give an example of a sequence (a_n) such that (a_n) is monotonic increasing and bounded above.

29) Determine which of the following sequences are monotonic $(\log n)$.

30) A sequence cannot converge to two different limits.

10 Marks :

31) Write the first five terms of each of the following sequences $\left(\frac{\cos nx}{n^2+x^2}\right)$.

32) Write the first five terms of each of the following sequences $(n!)$.

33) Determine the range of the following sequences $(2n-1)$.

34) Write the first five terms of each of the following sequences

$$f(n) = \begin{cases} n \; if \; n \; is \; odd \\ \frac{1}{n} \; if \; n \; is \; even \end{cases}$$

35) Determine the range of the following sequences The constant sequence $a, a, a, \ldots$.

36) Give examples of sequences (a_n) Such that

i) (a_n) is bounded above but not bounded below.

ii) (a_n) is bounded below but not bounded above.

iii) (a_n) is a bounded sequence.

iv) (a_n) is neither bounded above nor bounded below.

37) Determine the l.u.b and g.l.b of the following sequences if $1, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{4}}, \ldots, \frac{1}{\sqrt{n}}, \ldots$

38) Determine which of the following sequences are monotonic $((-1)^{n+1}n)$

39) If (a_n) is monotonic increasing show that $(\lambda \, a_n)$ is increasing if λ is positive and $(\lambda \, a_n)$ is decreasing if λ is negative.

40) Prove that

i) $\lim_{n\to\infty} \frac{1}{n^2} = 0$.

ii) $\lim_{n\to\infty} \left(1 + \frac{1}{n!}\right) = 1$.

UNIT - II

Choose the correct Answer :

1) A sequence $\{a_n\}_{n=1}^{\infty}$ is bounded iff there is a real number S such that …………..
   a) $|a_n| \leq S$ for all n.
   b) $|a_n| \geq S$ for all n.
   c) $|a_n| = S$ for all n.
   d) None of these.
2) A sequence $\{a_n\}_{n=1}^{\infty}$ is bounded from below if for real number R ………..
   a) $a_n \leq R$ for all n.
   b) $a_n \geq R$ for all n.
   c) $a_n = R$ for all n.
   d) None of these.
3) If $\{a_n\}_{n=1}^{\infty}$ converges to A and B both then ………..
   a) $A > B$
   b) $A = B$
   c) $A \leq B$
   d) None of these
4) A monotonic increasing sequence which is not bounded above…………
   a) Diverges to ∞
   b) Diverges to $-\infty$
   c) Converges to its l.u.b.
   d) Converges to its g.l.b.
5) A monotonic decreasing sequence which is ………. Converges to its g.l.b.
   a) Bounded above.
   b) Bounded below.
   c) Not bounded above.
   d) Not bounded below.
6) Evaluate the limits of the following sequence as $n \to \infty$ if $\left(\sqrt{(n^2+n-n)}\right)$
   a) 1
   b) $3/2$
   c) $1/2$

d) $4/5$

7) Evaluate the limits of the following sequence as $n \to \infty$ if $((-1)^n/n)$

a) ∞

b) -1

c) 1

d) 0

8) A sequence $\{a_n\}_{n=1}^{\infty}$ is bounded from above if for real number R ………..

a) $a_n \geq R$ for all n.

b) $a_n \leq R$ for all n.

c) $a_n = R$ for all n.

d) None of these.

9) Let $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ be two sequence such that $\{a_n\}_{n=0}^{\infty}$ and $\{a_n b_n\}_{n=0}^{\infty}$ converges respectively to A and AB, then $\{b_n\}_{n=0}^{\infty}$ converges iff………

a) $A \neq 0$

b) $A = 0$

c) $B = 0$

d) None of these.

10) If $\{b_n\}_{n=1}^{\infty}$ is an increasing bounded sequence then for the sequence $\{b_n\}_{n=1}^{\infty}$ if following statement is false ………

a) $\{b_n\}_{n=1}^{\infty}$ is a convergent sequence.

b) $\{b_n\}_{n=1}^{\infty}$ is a divergent sequence.

c) $\{b_n\}_{n=1}^{\infty}$ is a monotonic sequence.

d) $\{b_n\}_{n=1}^{\infty}$ is a Cauchy sequence.

Answers :

1) a 2) a 3) b 4) a 5) b 6) c 7) d 8) a 9) a 10) b

2Marks :

11) If $(a_n) \to a$ and $k \in R$ then $(ka_n) \to ka$.

12) If $(a_n) \to a$ then $(|a_n|) \to |a_n|$.

13) If $(a_n) \to a$, $(b_n) \to b$ and $a_n \leq b_n$ for all n, then $a \leq b$.

14) Show that $\lim_{n\to\infty} \frac{n}{\sqrt{(n^2+1)}} = 1$.

15) Show that $\lim_{n\to\infty} \frac{\sin n}{n} = 0$.

16) Evaluate the limits of the following sequences as $n \to \infty$ if $\left(\frac{3n-4}{2n+7}\right)$.

17) Evaluate the limits of the following sequences as $n \to \infty$ if $\left(\frac{4-2n+6n^2}{7-6n+9n^2}\right)$.

18) Show that following sequence diverge to ∞ if $(n^3 + n^2 + n + 1)$.

19) Prove that $\left(\frac{n!}{n^n}\right)$ converges.

20) Verify whether the following sequences are monotonic and discuss their behavior $\left(\frac{2n-7}{3n+2}\right)$.

5 Marks :

21) If $(a_n) \to a$ and $(b_n) \to b$ then $(a_n b_n) \to ab$.

22) If $(a_n) \to a$ and $a_n \geq 0$ for all n then $a \geq 0$.

23) If $(a_n) \to l$ and $(b_n) \to l$ and $a_n \leq c_n \leq b_n$ for all n, then $(c_n) \to l$.

24) If $(a_n) \to \infty$ and $(b_n) \to \infty$ then $(a_n + b_n) \to \infty$.

25) If $(a_n) \to \infty$ and $(b_n) \to \infty$ then $(a_n b_n) \to \infty$.

26) Show that $\lim_{n\to\infty} \left(\frac{1^2+2^2+\cdots+n^2}{n^3}\right) = \frac{1}{3}$.

27) Show that $\lim_{n\to\infty} \left(\frac{1}{\sqrt{(2n^2+1)}} + \frac{1}{\sqrt{(2n^2+2)}} + \cdots + \frac{1}{\sqrt{(2n^2+n)}}\right) = \frac{1}{\sqrt{2}}$.

28) If $(a_n) \to -\infty$ and $(b_n) \to -\infty$ then show that $(a_n + b_n) \to -\infty$ and $(a_n b_n) \to \infty$.

29) Let $a_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{n!}$. show that $\lim_{n\to\infty} a_n$ exists and lies between 2 and 3.

30) Let $a_n = 1 + \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n}$. Show that (a_n) converges.

10 Marks :

31) If $(a_n) \to a$ and $(b_n) \to b$ then $(a_n + b_n) \to a + b$ and $(a_n - b_n) \to a - b$.

32) If $(a_n) \to a$ and $a_n \neq 0$ for all n and $a \neq 0$, then $\left(\frac{1}{a_n}\right) \to \frac{1}{a}$.

33) If $(a_n) \to a$ and $a_n \geq 0$ for all n and $a \neq 0$, then $\left(\sqrt{a_n}\right) \to \sqrt{a}$.

34) Let $(a_n) \to \infty$. Then

i) if $c > 0, (c\, a_n) \to \infty$.

ii) if $c < 0, (c\, a_n) \to -\infty$.

35) Show that $\lim_{n\to\infty} \frac{3n^2+2n+5}{6n^2+4n+7} = \frac{1}{2}$.

36) Show that $\lim_{n\to\infty} \left(a^{1/n}\right) = 1$ where $a > 0$ is any real number.

37) Show that $\lim_{n\to\infty} \left(n^{1/n}\right) = 1$.

38) If $(a_n) \to -\infty$, then show that $(ka_n) \to -\infty$ if $k > 0$ and $(ka_n) \to \infty$ if $k < 0$.

39) Show that the sequence $\left(1 + \frac{1}{n}\right)^n$ converges.

40) Discuss the behavior of the geometric sequence (r^n).

UNIT - III

Choose the correct Answer :

1) If a sequence is not a Cauchy sequence, then it is a ……..
   a) Divergent sequence
   b) Convergent sequence
   c) Bounded sequence
   d) None of these
2) A: Every convergent sequence is Cauchy sequence.
   B: Every Cauchy sequence is a convergent sequence.
   a) A and B both are false
   b) A is true
   c) B is true
   d) A and b both true
3) Every Cauchy sequence is ……..
   a) Unbounded sequence
   b) Bounded sequence
   c) Divergent sequence
   d) None of these
4) Let $\{a_n\}_{n=1}^{\infty}$ be a sequence converges to 0 and $\{b_n\}_{n=1}^{\infty}$ be a sequence that is bounded, then $\{a_n b_n\}_{n=1}^{\infty}$ is a sequence that ……..
   a) Converges to one
   b) Converges to zero
   c) Is divergent sequence
   d) None of these

5) Let sequence $\{a_n\}_{n=1}^{\infty}$ converges to A and sequence $\{b_n\}_{n=1}^{\infty}$ converges to B, with $a_n \leq b_n$ for all n, then ……..

a) $A \leq B$

b) $A = B$

c) $A \geq B$

d) None of these

6) Let sequence $\{a_n\}_{n=1}^{\infty}$ converges to A and sequence $\{b_n\}_{n=1}^{\infty}$ converges to A and B respectively, then $\left\{{}^{a_n}/_{b_n}\right\}_{n=1}^{\infty}$ converges to ${}^{A}/_{B}$ if ………

a) $b_n \neq 0$ for all n and $B = 0$

b) $b_n \neq$ for some n

c) $b_n \neq 0$ for all n and $B \neq 0$

d) None of these

7) If $\{a_n\}_{n=1}^{\infty}$ is decreasing and bounded, then $\{a_n\}_{n=1}^{\infty}$……..

a) Convergent sequence

b) Divergent sequence

c) Non-Cauchy sequence

d) None of these

8) Let (a_n) be a sequence. Let (n_k) be a strictly increasing sequence of natural number. Then (a_{n_k}) is called a ……….. of (a_n).

a) Sequence

b) Subsequence

c) Bounded

d) None of these

9) Every sequence (a_n) has a ………..

a) Monotonic sequence

b) Sub sequence

c) Sequence

d) None of these

10) Find all the limit points of each of the following sequence $({}^{1}/_{n})$

a) ∞

b) $-\infty$

c) 1

d) 0

Answers :

1) a 2) d 3) b 4) b 5) a 6) c 7) a 8) b 9) a 10) d

2 Marks :

11) Statement of Cauchy's second limit theorem.

12) Show that $\lim_{n\to\infty} 1/n(1+\frac{1}{2}+\cdots+\frac{1}{n})=0$

13) Show that $\lim_{n\to\infty} n^{1/n}=1$

14) Define Sub sequence.

15) Define Peak point.

16) Define Limit points.

17) Define Cauchy sequences.

18) Any convergent sequence is a Cauchy sequence.

19) Any sequence (a_n) is a subsequence of itself.

20) Find all the limit point of (n^2).

5 Marks :

21) If a sequence (a_n) is converges to l, then every subsequence (a_{n_k}) of an also converges to l.

22) If the subsequences (a_{2n-1}) and (a_{2n}) of a sequences a_n. Converge to the same limit l, then a_n also converges to l.

23) Every bounded sequence as a convergence subsequence.

24) Every bounded sequence has at least one limit point.

25) Any Cauchy sequence is a bounded sequence.

26) Let (a_n) be a Cauchy sequence. If (a_n) has a subsequence (a_{n_k}) converging to l, then $(a_n)\to l$.

27) A sequence (a_n) in R is convergent iff it is a Cauchy's sequence.

28) Evaluate the limits of the following sequence whose n^{th} terms are given $\frac{1}{n}(1 + 2^{1/2} + 3^{1/3} + \cdots + n^{1/n})$.

29) Construct a sequence having exactly 10 limit points..

30) Find all the limit point of $(2n - 1)$.

10 Marks :

31) State and prove Cauchy's first limit theorem.

32) State and prove Cesaro's theorem.

33) State and prove Cauchy's second limit theorem.

34) Let (a_n) be any sequence and $\lim_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right| = l$. If $l > 1$ then $(a_n) \to 0$.

35) If the sequences a_n and $b_n \to 0$ and b_n is strictly monotonic decreasing then $\lim_{n\to\infty} (\frac{a_n}{b_n}) = \lim_{n\to\infty} (\frac{a_n - a_{n+1}}{b_n - b_{n+1}})$ provided the limit on the right hand side exists whether finite or infinite.

36) Prove that $\frac{1}{n}[(n+1)(n+2) \ldots (n+n)]^{1/n} \to 4/\mathrm{e}$

37) Let (a_n) be a sequence. A real number 'a' is a limit point of (a_n) iff there exists a sub sequence (a_{n_k}) of (a_n) converging to 'a'.

38) A sequence (a_n) converges to l iff (a_n) is bounded and l is the only limit point of the sequence.

39) Every sequence (a_n) has a monotonic subsequence.

40) Show that a sequence (a_n) diverges to ∞ iff ∞ is the only limit point of (a_n).

UNIT - IV

Choose the correct Answer :

1) Determine whether the sequence defined by $a_n = In(2n^3 + 2) - In(5n^3 + 2n^2 + 4)$ converges or diverges. If it converges, find the limit.

a) 0

b) $In\left(\frac{2}{5}\right)$

c) $-In\left(\frac{2}{5}\right)$

d) 2

2) Find all possible value of x for which the series $\sum \frac{9+x^n}{5^n}$ converges.

a) It is not possible to find such x because the series diverges.

b) $x > 0$

c) $|x| < 1$

d) $-5 < x < 5$

3) Determine whether the sequence defined by $a_n = n^2 \, cos\left(\frac{2}{n^2} + \frac{\pi}{2}\right)$ converges or diverges. If it converges, find the limit.

a) -2

b) -1

c) 1

d) 2

4) Which of the following sequence converge?

$a_n = \frac{(2n+1)!}{(n+4)!}$, $b_n = \frac{\pi^n}{n^{100}}$, $c_n = \frac{In(n^{10})}{\sqrt{n}}$, $d_n = \frac{n^4}{(n+1)!}$

a) $\{d_n\}$ only

b) $\{a_n\}, \{b_n\}$ only

c) $\{c_n\}, \{d_n\}$ only

d) $\{a_n\}, \{d_n\}$ only

5) Assume the terms of a sequence) $\{a_n\}$ are given by the following formula $a_n = \frac{1}{3n^3} + \frac{2^2}{3n^3} + \frac{3^2}{3n^3} + \cdots + \frac{n^2}{3n^3}$ Find the limit of the sequence or conclude that it diverges.

a) 0

b) 1

c) $\frac{1}{9}$

d) $\frac{1}{6}$

6) Determine the value of series $\sum_{n=0}^{\infty} \frac{2^{n-2}+3^{n+1}}{4^n}$ or conclude that it diverges.

a) $\frac{25}{2}$

b) $\frac{97}{8}$

c) $\frac{13}{2}$

d) 4

7) Assume $\sum_{n=1}^{\infty} a_n$ is an infinite series with partial sums given by $S_N = 4 + \frac{2}{N}$. What is a_5?

a) $\frac{1}{10}$

b) $-\frac{1}{10}$

c) $\frac{3}{10}$

d) $-\frac{3}{10}$

8) The sequence $\left\{\frac{n}{n+1}\right\}$ is …….

a) Increasing sequence

b) Decreasing sequence

c) Unbounded

d) None of these

9) The sequence $\left\{\frac{(-1)^n}{n}\right\}$ is ………..

a) Unbounded

b) Decreasing

c) Increasing

d) None of these

10) The sequence $\left\{a + \frac{(-1)^n b}{n}\right\}$…………

a) Bounded

b) Unbounded

c) Increasing

d) None of these

Answers :

1) b 2) d 3) a 4) c 5) c 6) a 7) b 8) a 9) a 10) a

2 Marks :

11) Define Infinite series.
12) Let $\sum a_n$ be a convergent series converging to the sum s. Then $\lim_{n\to\infty} a_n = 0$.
13) Discuss the convergence of the series $\sum \frac{1}{\sqrt{n^3+1}}$.
14) Discuss the convergence of the series $1 + \frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \cdots$
15) Discuss the convergence of the series $\sum_3^\infty (\log logn)^{-logn}$.
16) Show that the series $\sum(\frac{1}{2^n})$ converges to the sum 1.
17) Show that the series $1 + 2 + 3 + \cdots$ diverges to ∞.
18) Discuss the convergence of the following series whose n^{th} terms are given $\frac{5+n}{3+n^2}$.
19) Discuss the convergence of the following series whose n^{th} terms are given $\frac{2n}{n^2+1}$.
20) Discuss the convergence of the following series whose n^{th} terms are given $\frac{\sqrt{n}}{n^2-1}$.

5 Marks :

21) Let $\sum a_n$ converge to a and $\sum b_n$ converge to b then $(a_n \pm b_n) \to a \pm b$ and $\sum ka_n \to ka$.
22) Apply Cauchy's general principle of convergent . To show the series $\sum(1/n)$ is not convergent.
23) Applying Cauchy's general principle of convergent. Prove that $1 - 1/2 + 1/3 - \cdots + (-1)^n/n + \cdots$ is convergent.
24) Discuss the convergence of the series $\sum \frac{1^2+2^2+\cdots+n^2}{n^4+1}$.
25) Discuss the convergence of the following series whose n^{th} terms are given $\frac{n^4-5n^2+1}{n^6+3n^2+2}$.
26) Discuss the convergence of the following series whose n^{th} terms are given $\frac{1}{n\sqrt{(n^2+1)}}$.
27) Discuss the convergence of the following series whose n^{th} terms are given $\frac{n}{(n^2+1)^{2/3}}$.

28) Prove that if $\sum c_n$ is a convergent series of positive terms then so is $\sum a_n c_n$ where (a_n) is a bounded sequence of positive terms is.

29) Show that if $\sum a_n$ converges and $\sum b_n$ diverges then $\sum(a_n + b_n)$ diverges.

30) Use the inequality $e^x > x$ if $x > 0$ to show that the series $\sum e^{-n^2}$ convergent.

10 Marks :

31) State and prove Cauchy's general principal of convergence.

32) i) Let $\sum c_n$ be a convergent series of positive terms. Let $\sum a_n$ be another series of positive terms. If there exists $m \in N$ such that $a_n \leq c_n$ for all $n \geq m$ then $\sum a_n$ is also convergent.

ii) Let $\sum d_n$ be a divergent series of positive terms. Let $\sum a_n$ be another series of positive terms. If there exists $m \in N$ such that $a_n \geq d_n$ for all $n \geq m$ then $\sum a_n$ is also divergent.

33) i) If $\sum c_n$ converges and if $\lim_{n\to\infty}(\frac{a_n}{c_n})$ exists and is finite then $\sum a_n$ also converges.

ii) If $\sum d_n$ diverges and if $\lim_{n\to\infty}(\frac{a_n}{d_n})$ exists and is greater than zero then $\sum a_n$ diverges.

34) i) Let $\sum c_n$ be a convergent series of positive terms. Let $\sum a_n$ be another series of positive terms. If there exists $m \in N$ such that $\frac{a_{n+1}}{a_n} \leq \frac{c_{n+1}}{c_n}$ for all $n \geq m$, then $\sum a_n$ is convergent.

ii) Let $\sum d_n$ be a divergent series of positive terms. Let $\sum a_n$ be another series of positive terms. If there exists $m \in N$ such that $\frac{a_{n+1}}{a_n} \geq \frac{d_{n+1}}{d_n}$ for all $n \geq m$, then $\sum a_n$ is divergent.

35) The harmonic series $\sum \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

36) Discuss the convergence of the following series whose n^{th} terms are given $\frac{n}{(n^2+1)^{3/2}}$.

37) Show that $\sum \frac{1}{4n^2-1} = \frac{1}{2}$.

38) Prove that a sequence (a_n) is convergent iff $\sum(a_{n+1} - a_n)$ is convergent.

39) Let a and b be two positive real numbers. Show that the series $a + b + a^2 + b^2 + a^3 + b^3 + \ldots$ converges if both a and $b < 1$ and diverges if either $a \geq 1$ or $b \geq 1$.

40) Discuss the convergence of the following series whose n^{th} terms are given $\frac{n(n+1)}{(n+2)(n+3)(n+4)}$

UNIT - V

Choose the correct Answer :

1) The series $\sum_{n=1}^{\infty} \frac{(-1)^n n^{500}}{(1.0001)^n}$

a) Converges absolutely.

b) Converges conditionally, but not absolutely.

c) Converges to $+\infty$

d) Converges to $-\infty$

2) The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} \left(1 + \frac{1}{n^2}\right)$.

a) Is bounded but divergent.

b) Converges absolutely.

c) Converges conditionally, but not absolutely.

d) Converges to $+\infty$

3) Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{(-1)^n}{1+\frac{1}{n}}$ II. $\sum_{n=2}^{\infty} \frac{1}{In(n^4)}$ III. $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$

a) None of them

b) I and III

c) I and II

d) II and III

4) The series $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$. Find the value of the series $\sum_{n=2}^{\infty} \left(\frac{2}{n}\right)^4$.

a) $\frac{8\pi^4}{45}$

b) $16\left(\frac{\pi^4}{90} - 1\right)$

c) 16

d) -2

5) Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{2^n+n^4}{4^n+n^2}$ II. $\sum_{n=1}^{\infty} \frac{4^n}{5^n+n}$

a) I only

b) II only

c) I and II

d) None of these

6) Which of the following alternating series converge conditionally, but not absolutely?

I. $\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n} In\, n}$ II. $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(In\, n)^2}$ III. $\sum_{n=2}^{\infty} \frac{cos(\pi n)}{2^{n-3}}$

a) None of them

b) I only

c) II only

d) III only

7) For which values of p does the series $\sum_{n=1}^{\infty} \frac{e^n}{(2+e^{2n})^p}$ converge?

a) All values of p

b) $-1 < p < 1$

c) $p > 1$

d) $p > \frac{1}{2}$

8) Let $\sum_{n=1}^{\infty} a_n$ be a series with partial sumsS_N. Which of the following statements are always true?

I. If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

II. If $\sum_{n=1}^{\infty} a_n = L$, then $\lim_{n\to\infty} a_n = L$.

III.If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

IV.If $\lim_{N\to\infty} S_N = L$, then $\sum_{n=1}^{\infty} a_n = L$.

a) I and II

b) I and III

c) III and IV

d) II and III

9) The sequence $\{1,0,1,0,\ldots\}$ is ………

a) Increasing

b) Decreasing

c) Bounded

d) None of these

10) The series $\sum_{n=1}^{\infty} \frac{1}{n^n}$ is ………

a) Convergent

b) Divergent

c) Oscillatory

d) None of these

Answers :

1) a 2) c 3) a 4) b 5) c 6) b 7) d 8) c 9) c 10) a

2 Marks :

11) Define D'Alembert's ratio test.

12) Statement of Cauchy's root test.

13) Statement of Leibnitz test.

14) Test the convergence of $\sum \frac{(-1)^n \sin n \infty}{n^3}$.

15) Define conditionally convergent.

16) Test the convergent of $\sum \frac{1}{n \log n}$.

17) Test the convergence of the series $\sum \frac{1}{n(\log n)^p}$.

18) Define Alternating series.

19) Show that the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$ converges.

20) Show that series $\sum (-1)^{n+1} \frac{n}{3n-2}$ oscillates.

5 Marks :

21) Test the convergence $\sum \frac{n^n}{n!}$

22) Test the convergence of the series $\sum \frac{3^n n!}{n^n}$

23) Test the convergence of the series $\sum \frac{x^n}{n}$

24) Test the convergence of the series $\sum \frac{n^p}{n!}$ $(p > 0)$.

25) Test the convergence of $\sum \frac{1}{(\log n)^n}$.

26) Show that the series $\sum \frac{(-1)^{n+1}}{\log(n+1)}$ converges.

27) Show that series $\sum \frac{(-1)^{n+1}}{\log(n+1)}$ converges.

28) Show that following series converges $\frac{1}{2^3} - \frac{1}{3^3}(1+2) + \frac{1}{4^3}(1+2+3) - \frac{1}{5^3}(1+2+3+4) + \cdots$

29) Any absolutely convergent series is convergent.

30) Show that the series $\sum \frac{x^{n-1}}{(n-1)!}$ converges absolutely for all value of x .

10 marks :

31) Test the convergence of the series $\frac{1}{3} + \frac{1.2}{3.5} + \frac{1.2.3}{3.5.7} + \cdots$

32) Test the convergence of the series $\sum \frac{2^n n!}{n^n}$.

33) Test the convergence of the series $\sum \sqrt{\frac{n}{n+1}}.x^n$ where x is any positive real number.

34) Test the convergence of the series $\sum \frac{n^2+1}{5^n}$.

35) State and prove Cauchy's root test.

36) Prove that the series $\sum e^{-\sqrt{n}}\ x^n$ converges if $0 < x < 1$ and diverges if $x > 1$.

37) Test the convergence of $\sum \frac{n^3+a}{2^n+a}$.

38) Test the convergence of series $\frac{1}{2} + \frac{1}{3} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^3} + \cdots$

39) State and prove Leibnitz test.

40) Test for convergence of the series $\sum \frac{(-1)^n}{n^p}$.

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