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QUESTION BANK

Title of the Paper

STATICS

COURSE – III B.Sc., Maths

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CORE COURSE XI

STATICS

OBJECTIVE:

1.To provide the basic knowledge of equilibrium of a particle.

2. To develop a working knowledge to handle practical problems.

UNIT I :

Introduction –Forces acting at a point: Triangle of forces –Resolution of force –Condition of equilibrium.

UNIT II :

Parallel forces and Moments: Resultant of parallel forces –Theorems on Moments –Moment about an axis –couples.

UNIT III :

Equilibrium of three forces acting on a rigid body: Conditions of equilibrium –Trigonometrical theorems and problems -Coplanar forces: Reduction of Coplanar forces–Equation of Line of action of the resultant–Conditions of equilibrium

UNIT IV :

Friction: Introduction –Laws of Friction –Definitions –Equilibrium of a particle on a rough inclined plane.

UNIT V :

Equilibrium of strings: Equation of the Common Catenary -Parabolic Catenary.

UNIT – I

CHOOSE THE CORRCET ANSWER:

1.If the resultant at forces 3p, 5p is equal to 7p. Find the angle between the forces

a) 45^0

b) 60^0

c) 30^0

d) 50^0

2. If m is the mass of the particle and $\vec{v}$ its velocity, then m$\vec{v}$ is called___?

a) Linear momentum

b) Tension

c) Reaction

d) None of these

3. When the bodies tend to approach each other the forces exerted is called __?

a) Reaction

b) Repulsion

c) Attraction

d) Tension

4. When the bodies tend to separate out forces exerted is called ___?

a) Repulsion

b) Attraction

c) Reaction

d) None of these

5. If the resultant of two forces acting at a point with the magnitude 7, 8 is force with a magnitude 13. Find the angle between the two given forces?

a) 30^0

b) 40^0

c) 50^0

d) 60^0

6. Two forces are given magnitude P and Q act at a point at an angle α, if α = 0^0 what will be the maximum value of the resultant.

a) $P + Q$

b) $P - Q$

c) $Q + P$

d) $Q - P$

7. Two forces are given magnitude P and Q act at a point at an angle α, if α = 180^0 what will be the minimum value of the resultant.

a) $P + Q$

b) $P - Q$

c) $Q + P$

d) $Q - P$

8. The science which deals with the conditions for lack of motion under given forces is called ___?

a) Mechanics

b) Dynamics

c) Statics

d) None of these

9. When the number of forces act on a body and keep it at rest the force are said to be___?

a) Tension

b) Reaction

c) Attraction

d) Equilibrium

10. The science which deals with the motion of particles or bodies under the influence of forces are called __?

a) Mechanics

b) Dynamics

c) Statics

d) None of these

ANSWERS:

1) b 2) a 3) c 4) a 5) d 6) a 7) b 8) c 9) d 10) b

TWO MARK QUESTIONS

11. Define Statics.

12. Define Force.

13. Define Linear momentum.

14. Define Equilibrium.

15. Define Equilibrium of two forces.

16. Let P and Q be the given two forces R is resultant and θ is the angle between forces then find the following p = 8, q = 7, $\theta = 60^0$, R = ?

17. Define Conditions of equilibrium.

18. Define Graphical condition.

19. Define Analytical condition.

20. State the lami's theorem.

FIVE MARK QUESTIONS

21. State and prove parallelogram law of theorem.

22. The resultant of two forces P and Q is R. If Q is doubled then R is doubled. If Q is reverse then also R is doubled. Show that P: Q: R = $\sqrt{2}:\sqrt{3}:\sqrt{2}$.

23. Two forces are given magnitude P and Q act at a point at an angle α what will be the maximum and minimum value of the resultant.

24. State and prove triangle law of forces.

25. The resultant of two forces P, Q is of magnitude P. Show that if P is doubled the new resultant is perpendicular to the force Q and is magnitude is $\sqrt{4P^2 - Q^2}$.

26. State and prove polygon law of forces.

27. A string ABC has its end tied two fixed point A and B in the same horizontal line at a point C. A weight w is attached. Find tension in the string CA are $\frac{wb}{4c\Delta}(c^2 + a^2 - b^2)$.

28. The algebraic sum of the resolved parts of the forces in any direction is equal to the resolved part of the resultant in the same direction.

29. Forces of magnitude P, Q, R at along HA, HB, HC respectively are in equilibrium. Where H is the orthocenter of triangle ABC. Show that P : Q : R = a : b : c.

30. OA, OB, OC are the lines of action of two forces P, Q and there resultant R respectively any transversal meets in the line l, m, n respectively. Prove that $\frac{P}{OL} + \frac{Q}{OM} = \frac{R}{ON}$.

TEN MARK QUESTIONS

31. State and prove parallelogram law of forces.

32. The resultant of two forces P, Q acting at a certain angle is x and that of P, R acting at the same angle is also X. The resultant of Q, R again acting at the same angle is Y. Prove that P = $(X^2 + QR)^{\frac{1}{2}}$ = QR $(Q + R)/Q^2 + R^2 - Y^2$. Prove also that if P + Q + R = 0. Then Y = X.

33. The resultant of two forces P, Q acting at an angle θ is equal to $(2m + 1)\sqrt{P^2 + Q^2}$ when they act the angle 90^0- θ the resultant is $(2m - 1)\sqrt{P^2 + Q^2}$. Then prove that $tan\theta = \frac{m-1}{m+1}$.

34. If the resultant at forces 3p, 5p is equal to 7p.

(i) Find the angle between the forces.

(ii) The angle which the resultant makes with the first forces.

35. State and prove Lami's theorem.

36. Two force of magnitude $k\ cosA$, $k\ cosB$ act along CA, CB of a triangle ABC. Prove that the resultant is $k\ sinC$.

37. ABC are the verticals of a given triangle ABC forces PQR acting along the line OA, OB, OC are in equilibrium. Where O is the circumcentre of the circum scribed circle of the triangle. Prove that $P:Q:R = a^2(b^2+c^2-a^2):b^2(c^2+a^2-b^2):c^2(a^2+b^2-c^2)$.

38. If I is the incentre of a triangle ABC and the forces P, Q, R acting along IA, IB, IC are in equilibrium. Show that triangle ABC is an equilateral.

39. A weight is supported on a smooth plane of inclination α by string inclined to the horizon at an angle γ. If the slope of the plane increased to β and the slope of the string unaltered the tension of the string is doubled. Prove that $cot\alpha - 2cot\beta = tan\gamma$.

40. ABCDEF is a regular hexagon and at A, act forces represented by $\overrightarrow{AB}$, 2$\overrightarrow{AC}$, 3$\overrightarrow{AD}$, 4$\overrightarrow{AE}$ and 5$\overrightarrow{AF}$. Show that the magnitude of the resultant is AB$\sqrt{351}$ and that is makes an angle $\tan^{-1}\left(\frac{7}{\sqrt{3}}\right)$ with AB.

UNIT – II

CHOOSE THE CORRECT ANSWER:

1.Two parallel forces are said to be ___ when they act in the same direction.

a) Unlike

b) Like

c) a and b

d) None of these

2. Two parallel forces are said to be ___ when they act in the opposite parallel direction.

a) Unlike

b) Like

c) a and b

d) None of these

3. The resultant of a system of parallel forces of given magnitude, acting at given points of a body, will always pass through a fixed point, for all direction of parallelism. This point is called __?

a) Parallel forces

b) Unlike parallel forces

c) Centre of parallel force

d) None of these

4. If a system of forces acting on a body keep it in equilibrium, the algebraic sum of their moments about any line in the body is __?

a) 0

b) 1

c) 2

d) 3

5. Two equal and unlike parallel forces not acting at the same point are said to constitute a __?

a) Arm of couple

b) Coplanar couple

c) Couple

d) None of these

6. The algebraic sum measures the total turning effect of the forces of the couple upon the body and is called the__?

a) Arm of couple

b) Coplanar couple

c) Couple

d) Moment of the couple

7. The algebraic sum of the moments of any number of coplanar forces about any point on the line of action of their resultant is __?

a) 0

b) 1

c) 2

d) 3

8. If a force is represented completely by a straight line, its moment about any point is given by__ the area of the triangle which the straight line subtends at that point.

a) Once

b) Twice

c) Thrice

d) None of these

9. The magnitude of the resultant of two like parallel force is their ___?

a) Divide

b) Difference

c) Sum

d) None of these

10. The magnitude of the resultant of two unlike parallel force is their ___?

a) Sum

b) Difference

c) Divide

d) None of these

ANSWERS:

1) b 2) a 3) c 4) a 5) c 6) d 7) a 8) b 9) c 10) b.

TWO MARK QUESTIONS

11. Define Parallel forces.

12. Define Couples.

13. Define Moment of couple.

14. Define Arm and axis of a couple.

15. Write state the varignon's theorem.

16. Write the formula for position vector.

17. Three like parallel forces P, Q, R act at the vertices of a triangle ABC. Show that their resultant passes through the centroid if P = Q= R.

18. Define Unlike parallel forces.

19. Define Centre of two parallel forces.

20. Write state the principle of moment.

FIVE MARK QUESTIONS

21. If two like parallel forces of magnitude P, Q acting on a rigid body at A, B are interchanged in position. Show that the line of action of the resultant displaced through distance $\frac{AB(P-Q)}{P+Q}$.

22. If two like parallel forces of magnitude P, Q acting on a rigid body at A, B. If the second force moved away from the first parallely through a distance d. Show that the resultant of force moves through a distance $\frac{dQ}{P+Q}$.

23. Three like parallel forces P, Q, R act at the vertices of a triangle ABC. Show that their resultant passes through

    (i) The centroid if P = Q = R

    (ii) The in centre if $\frac{P}{a} = \frac{Q}{b} = \frac{R}{c}$.

24.Three like parallel forces P, Q, R act at the vertices of a triangle ABC. If the resultant passes through the orthocenter O. Show that $\frac{P}{\tan A} = \frac{Q}{\tan B} = \frac{R}{\tan C}$.

25. Prove that a system of coplanar forces reduce either to a single force or to a couple.

26. Prove that if two couples where moments are equal and opposite act in the opposite plane upon a rigid body they balance one another.

27. Show that a couple can be transformed to a plane parallel to its own plane without altering its effect on the rigid body on which its acting.

28. Forces p, 4p, 2p, 6p act along the side AB, BC, CD, DA of a square ABCD of side a. Find the magnitude of the resultant is the $p\sqrt{5}$ and prove that the equation of the line of action referred to AB, AD as the XY axis is 2x – y + 6a = 0.

29. Forces with components (1, 0), (-2, 0), (1, 1) act respectively at the point (0, 0), (1, 1), (1, 0) what is the system equivalent to?

30. Two like parallel force of magnitude P, Q acting on a rigid body if Q is changed $\frac{P^2}{Q}$ with the line of action being the same. Show that the line of action of the resultant will be the same as it would be if the forces where simply interchanged.

TEN MARK QUESTIONS

31. To find the resultant of two parallel forces acting on a rigid body.

32. State and prove varignon's theorem.

33.P, Q, R are three forces along the sides BC, CA, AB of a triangle ABC taken in order. Show that if there resultant

(i) passes through in centre then $P + Q + R = 0$

(ii) passes through the centroid then $\frac{P}{a} + \frac{Q}{b} + \frac{R}{c} = 0$

(iii) passes through the circumcentre then P cosA + Q cosB + R cos C = 0.

34. Three forces P, Q, R act along the sides BC, CA, AB of triangle ABC. Show that the resultant passes through

(i) The incentre and circumcentre $\frac{P}{\cos B - \cos C} = \frac{Q}{\cos C - \cos A} = \frac{R}{\cos A - \cos B}$

(ii) The circumcentre and orthocenter $\frac{P}{(b^2 - c^2)\cos A} = \frac{Q}{(c^2 - a^2)\cos B} = \frac{R}{(a^2 - b^2)\cos C}$

(iii) The orthocenter and centroid $\frac{P}{\sin 2A \sin(B-C)} = \frac{Q}{\sin 2B \sin(C-A)} = \frac{R}{\sin 2c \sin(A-B)}$

35. Show that a system of coplanar couples acting on a rigid body is equivalent to a couple in the some plane whose moment is equal to the sum of the moments of the given couples.

36. To show that a couple and force in the same plane reduce to a single force. This single force is the same as a given force, but has a different parallel line of action.

37. ABCDEF is a regular hexagon forces p, 2p, 3p, 2p, 5p, 6p act along AB, BC, DC, ED, EF, AF. Show that the six forces are equivalent to a couple and find the moment of the couple.

38. Forces P_1, P_2, P_3, P_4, P_5 and P_6 act along AB, BC, CD, DE, EF, FA of a regular hexagon. Show that they will be in equilibrium. If

(i) $P_1 – P_4 = P_3 – P_6 = P_5 – P_2$

(ii) $P_1 + P_2 + P_3 + P_4 + P_5 + P_6 = 0$.

39. The greatest resultant the two forces can have is of magnitude R and the least magnitude S. Show that they act at an angle 2α the magnitude of the resultant is $\sqrt{R^2 cos^2\alpha + S^2 sin^2\alpha}$.

40. A uniform plank AB of length 2a and weight w is supported horizontally on two horizontal pages C and D at a distance d apart. The greatest weight that can be placed at the two ends in succession without upsetting the plank are w_1 and w_2 respectively. Show that $\frac{w_1}{w+w_1} + \frac{w_2}{w+w_2} = \frac{d}{a}$.

UNIT – III

CHOOSE THE CORRECT ANSWER:

1.Three forces acting on a rigid body are in equilibrium, they must be__?

a) Coplanar

b) Parallel

c) Concurrent

d) None of these

2. The sum of the components of the forces along any line in the plane is__?

a) 0

b) 1

c) 2

d) 3

3. The sum of the moments of the forces about any one point in the plane is __?

a) 1

b) 2

c) 3

d) 0

4. If three coplanar forces acting on a rigid body keep it in equilibrium, they must either be concurrent or be all__?

a) Parallel

b) Not parallel

c) Perpendicular

d) None of these

5. A uniform rod of length 16a rests in equilibrium against a smooth vertical wall, and upon a peg at a distance ‘a’ from the wall. What is the inclination of the rod to the vertical is __?

a) 20^0

b) 30^0

c) 40^0

d) 50^0

6. Any system of forces acting in one plane on a rigid body can be reduced to a single force or a __?

a) Coplanar

b) Concurrent

c) Couple

d) None of these

7. The algebraic sum of the resolved parts in any one direction is not equal to __?

a) 1

b) 2

c) 0

d) 3

8. A system of forces will reduce to a single force if the algebraic sum of the moments of the forces about any point in their plane is __?

a) 0

b) 1

c) 2

d) 3

9. If the algebraic sum of the moments of a system of coplanar forces about each of three non – collinear points in their be the same in magnitude and sign, the system reduces to a __?

a) Couple

b) Coplanar

c) Concurrent

d) None of these

10. Forces p, 4p, 2p, 6p act along the sides AB, BC, CD, DA of a square of side a. Find the points where the resultant meets AB is __?

a) (-a, 0)

b) (-2a, 0)

c) (-3a, 0)

d) (-4a, 0)

ANSWERS:

1) a 2) a 3) d 4) a 5) b 6) c 7) c 8) a 9) a 10) c.

TWO MARK QUESTIONS

11. Write the cotangent formula.

12. Define Condition of equilibrium.

13. Define Coplanar forces.

14. What is the necessary and sufficient conditions that a system of coplanar forces acting on a rigid body.

15. Forces p, 4p, 2p, 6p act along the sides AB, BC, CD, DA of a square of side a. To show that reduce the system to a force at A and a couple.

16. Write the equation to the line of action of the resultant.

17. Forces 3p, 4p, 5p act respectively along the sides AB, BC, CA of an equilateral triangle ABC. Find the resultant.

18. Forces with components (1, 0), (-2, 0), (1, 1) act respectively at the points (0, 0), (1,1), (1, 0). What is the system equivalent to?

19. Forces with components (2, 0), (-1, 0), (1, 0) act respectively at the points (0, 0), (0,1), (1, 0). What is the system equivalent to?

20. When the three forces are concurrent?

FIVE MARK QUESTIONS

21. Show that three coplanar forces keep a rigid body in equilibrium, then either they all are parallel to one another or they are concurrent.

22. A solid cone of height h, and semi vertical angle α, is placed with its base against a smooth vertical wall and is supported by a string attached to its vertex and to a point in the wall. Show that the greatest possible length of the string is $l = h\sqrt{1 + \frac{16}{9} tan^2\alpha}$.

23. A rod AB rest within a smooth hemispherical bowl. The centre of gravity G divides if into two portions of length (a, b). Show that if 2α is the angle subtended by the rod at the centre of the bowl and θ is the inclination of the rod to the horizon in the equilibrium position, then $\tan\theta = \frac{b-a}{b+a}\, tan\alpha$.

24. A rod rests wholly within a smooth hemispherical bowl of radius r, its centre of gravity dividing the rod into two portions a and b. Show that if θ is the inclination of the rod to the horizontal in the position of equilibrium, then $\sin\theta = \frac{b-a}{2\sqrt{r^2-ab}}$.

25. Show that A set of sufficient conditions for a system of coplanar forces to coplanar forces to keep a rigid body in equilibrium is that the sums of moments of the forces about any two points A and B are individually zero and that the sum of the components of the forces in any direction not perpendicular to AB is zero.

26. A uniform rod of weight w is movable in a vertical plane about a hinge at one end, and at the other end is fastened weight $\frac{w}{2}$. The latter end is fastened by a string of length l to a point at a height c vertically over the hinge. Show that the tension of the string is $\frac{lw}{c}$.

27. A uniform bar AB of weight 2w and length l is free to turn about a smooth hinge at its upper end A, and a horizontal force applied to the end B keeps the bar in equilibrium with B at a distance 'a' from the vertical through A. Prove that the reaction at the hinge is equal to $w\sqrt{\frac{4l^2-3a^2}{l^2-a^2}}$.

28. A heavy sphere rests touching two smooth inclined planes one of which is inclined at 60^0 to the horizontal. If the pressure on this plane is one-half of the weight of the sphere, prove that the inclination of the other plane to the horizontal is 30^0.

29. A uniform rod AB rests with a point P on it (AP = 3/4) in contact with a fixed smooth peg and the end A attached to a light string which is fastened to a fixed point. If the rod makes an angle of 45^0 with the horizontal, show that the string makes with the horizontal an angle whose tangent is 2.

30. A uniform solid hemisphere of weight W rests with its curved surface on a smooth horizontal plane. A weight w is suspended from a point on the rim of the hemisphere. If the plane base of the rim is inclined to the horizontal at an angle θ, prove that $\tan\theta = \frac{8w}{3W}$.

TEN MARK QUESTIONS

31. A heavy uniform rod AB of length 2a rest partly within and partly without a fixed smooth hemispherical bowl of radius r. The rim of the bowl is horizontal and one point of the rod is in contact with the rim. If θ is the inclination of the rod to the horizon, show

that 2r cos2θ = a cosθ. Also show that the greatest possible inclination of the rod is $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

32. A solid hemisphere is supported by a string fixed to a point on its rim and to a point on a smooth vertical wall with which the curved surface of the hemisphere is in contact. If θ, $\emptyset$ are inclination of the string and the plane base on the hemisphere to the vertical. Prove that $\tan\emptyset = \frac{3}{8} + \tan\theta$.

33. A uniform rod AB of length 2a hinges against a smooth vertical wall being supported by a string of length 2l tied, to one end of the rod with the other end of the string being attached to a point c. In the wall above the rod show that the rod can rest inclined to the wall at an angle θ. Where $cos^2\theta = \frac{l^2-a^2}{3a^2}$.

34. A uniform beam of weight w hinged at one end is supported and the other end by a string so that the beam and the string are in a vertical plane and make the same angle α with the horizon. Show that the reaction R hinge and end tension T on the string are, $R = \frac{w}{4}\sqrt{8 + \ cosec^2\alpha}, T = \frac{w}{4}\ cosec\alpha.$

35. A square board rest in vertical plane with one corner against a smooth vertical wall the adjacent upper corner being attached to the wall by a string of same length as the side of a square. Show that if the string makes an angle θ with the vertical then $\tan\theta = \frac{1}{3}$.

36. A string of length 2l has one end attached to the extremity of a smooth heavy rod AB of length 2a and the other end carries a weightless ring C which slides on the rod. The string is hung over a smooth peg o. Show that, if θ is the angle which the rod makes with the vertical, then l cosθ = a $\sin^3\theta$.

37. A square plate ABCD of slide 2a and centre G is placed with its plane vertical between two smooth pegs P, Q which are in the same horizontal line at a distance c apart. Show that, in the position of equilibrium, the inclination of one of its edges to the horizontal is either 45^0 or $\frac{1}{2}\sin^{-1}\frac{a^2-c^2}{c^2}$.

38. A uniform beam of length l and weight w hanges from a fixed point by two strings of lengths a and b. Prove that the inclination θ of the rod to the horizon is given by $\sin\theta = \frac{a^2-b^2}{l\sqrt{2(a^2+b^2)-l^2}}$.

39. The altitude of a right cone is h and the radius of its base is a. A string of length l is fastened to the vertex and to a point on the circumference of the circular base and is

then put over a smooth peg. The cone rests with its axis horizontal. Show that the length of the string is $\sqrt{h^2 + 4a^2}$.

40. A sphere of radius a and weight w rests on a smooth vertical wall being suspended by a string of length l with one end attached to a point A on the surface of the sphere and the other end to a point B on the wall. Find the inclination θ of the string to the vertical and its tension in the position of equilibrium.

UNIT – IV

CHOOSE THE CORRECT ANSWER:

1.When one body in contact with another is in equilibrium, the friction exerted is just sufficient to maintain equilibrium and is called__?

a) Friction

b) Statical friction

c) Limiting friction

d) None of these

2. When one body is just on the point of sliding on another, the friction exerted attains its maximum value and is called___?

a) Friction

b) Statical friction

c) Limiting friction

d) Dynamical friction

3. When motion ensues by one body sliding over another the friction exerted is called__

a) Friction

b) Statical friction

c) Limiting friction

d) Dynamical friction

4. The ratio of the limiting friction to the normal reaction is called the __?

a) Coefficient of friction

b) Friction

c) Statical friction

d) Limiting friction

5. F be the friction and R the normal reaction between two bodies when equilibrium limiting is __?

a) $F = \mu R$

b) $R = F\mu$

c) $\mu = FR$

d) None of these

6. The angle of repose of a rough inclined plane is equal to the __?

a) Cone of friction

b) Angle of friction

c) Limiting friction

d) None of these

7. What is denoted by the angle of friction?

a) μ

b) μR

c) λ

d) λR

8. Two bodies act in perpendicular directions and they can be compounded into a single force. This single force is called the ___?

a) Normal reaction

b) Reaction

c) Resultant reaction

d) None of these

9. Tangential force between two bodies in contact prevents the one from sliding over the other such a force is called __?

a) Statical friction

b) Limiting friction

c) Dynamical friction

d) Force of friction

10. The cone of friction is

a) $\tan \mu$

b) $\tan^{-1} \mu$

c) $\tan \lambda$

d) $\tan^{-1} \lambda$

ANSWERS:

1) b 2) c 3) d 4) a 5) a 6) b 7) c 8) c 9) d 10) b.

TWO MARK QUESTIONS

11. Define Measuring force.

12. Define Earth gravitation.

13. Define Weight.

14. Write the Hooke's law.

15. Define Limiting friction.

16. Define Angle of friction.

17. Define Cone of friction.

18. Define Normal reaction and friction.

19. Define Reaction.

20. Write the laws of dynamic friction.

FIVE MARK QUESTIONS

21. Explain laws of friction.

22. Suppose a particle of weight ‘w’ lying s on a rough plane inclined at angle α to the horizontal is subject to the a force p along the plane in the upward direction if the equilibrium is limiting to find p.

23. A particle rest on a plane inclined at 45^0 to the horizontal being supported by a string along the line of the greatest slope. The ratio of the maximum and minimum tension consists with equilibrium is 2 : 1. Find the coefficient of friction.

24. A body of weight 4 kg rest in limiting equilibrium on a rough plane whose slope is 30^0. If the plane is raised to a slope of 60^0. To find the force along the plane required to support the body.

25. A uniform ladder AB rest in limiting equilibrium with the end A on a rough floor the coefficient of friction being μ and with the other end B against a smooth vertical wall. Show that if θ is the inclination the ladder to the vertical then $\tan\theta = 2\mu$ if 30^0. Find also μ.

26. Find the inclination θ to the vertical of a uniform ladder AB of length 2a and w weight which is in limiting equilibrium having contact with a rough horizontal floor and rough vertical wall the coefficient of friction.

27. A ladder which stands on a horizontal ground against a vertical wall has its centre of gravity at distance a and b from its lower and upper ends respectively. Show that if the ladder is in limiting equilibrium and if μ and μ' are coefficient of friction at the lower and upper contacts its inclination θ to be the vertical is given by $\tan\theta = \frac{(a+b)\mu}{(a-b)\mu'}$.

28. A uniform ladder of length l rest on a rough horizontal ground with its upper and projecting slightly over a smooth horizontal rod at a height h above the ground with the ladder is above to slip. Show that the coefficient of friction is equal to $\frac{h\sqrt{l^2-h^2}}{l^2+h^2}$.

29. A rod AB rest with in a fixed hemispherical bowl whose radius is equal to length of the rod. If μ is the coefficient of friction. Show that in limiting equilibrium the inclination θ of the rod to the horizontal is given by $\tan\theta = \frac{4\mu}{3-\mu^2}$.

30. A rod AB is supported at an inclination α to the horizontal with its lower end B on a rough horizontal plane by a string AC attached to A. Show that If μ is the coefficient of friction then the greatest inclination θ of the string to vertical is given by $\cot\theta = \frac{1}{\mu} + 2tan\alpha$.

TEN MARK QUESTIONS

31. Suppose a particle of mass is placed on a rough inclined plane at an angle α to the horizontal and force of magnitude s at on it a direction making an angle θ with the plane if the equilibrium is limiting to find s.

32. A solid hemisphere rest on a rough horizontal plane are against a smooth vertical wall. Show that, if the coefficient of friction μ is greater than $\frac{3}{8}$, then the hemisphere can rest in any position and if it is less the least angle that the base of the hemisphere can make with the vertical is $\cos^{-1}\left(\frac{8\mu}{3}\right)$.

33. A uniform ladder AB rest against a smooth wall at B and upon a rough ground at A. Boy whose weight is twice that of the ladder, climbs it. Prove that, the force of friction when he is at the top of the ladder is five times as great as when he is at the bottom.

34. A ladder is limiting equilibrium having contact with a rough horizontal floor and rough vertical wall whose coefficient of friction are μ, μ'. If θ is the inclination of the ladder to the vertical then, show that $\tan\theta = \frac{2\mu}{1-\mu\mu'}$. When $\mu = \mu'$, show that $\theta = 2\lambda$, where λ is the angle of friction.

35. A ladder is in equilibrium with one end resting on the ground and the other end against a vertical wall, if the ground and the wall are both rough, the coefficient of friction being μ and μ' and if the ladder is on the point of slipping, show that the inclination of the ladder to the horizon is given by $\tan\theta = \frac{1-\mu\mu'}{2\mu}$ when $\mu = \mu'$, show that $\theta = 90^0 - 2\lambda$, where λ is the angle of friction.

36. A ladder of lengthy 2a is in contact with a wall and a horizontal floor, the angle of friction being λ at each contact. If the centre of gravity of the ladder is at a distance ka below the midpoint, show that in the limiting equilibrium, the inclination θ to the vertical is given by $\cot\theta = \cot 2\lambda - k\ cosec\ 2\lambda$.

37. A ladder AB rest with A resting on the ground and B against a vertical wall, the coefficient of friction of the ground and the wall being μ and μ' respectively. The centre

of gravity G of the ladder divides AB in the ratio 1 : n. If the ladder is on the point of slipping at both ends, show that its inclination to the ground is by $\tan\theta = \frac{1-n\mu\mu'}{(n+1)\mu}$.

38. A square lamina rest with the ends of a side against a rough vertical wall and a rough horizontal ground. If the coefficient of friction for the ground and the wall are μ and μ', show that when the lamina is on the point of slipping, the inclination of the side in question to the horizontal is $\tan^{-1}\left[\frac{1-\mu\mu'}{1+2\mu+\mu\mu'}\right]$.

39. A solid hemisphere rest in equilibrium on a rough horizontal plane and against a rough vertical wall, the coefficient of friction being μ, μ'. Show that, if the equilibrium is limiting the inclination of the base to the vertical is $\cos^{-1}\left[\frac{8\mu(1+\mu')}{3(1+\mu\mu')}\right]$.

40. A solid hemisphere rest in equilibrium on a rough ground and against an equally rough vertical wall, the coefficient of friction being μ. Show that, if the equilibrium is limiting, the inclination of the base to the horizon is $\sin^{-1}\left[\frac{8\mu(1+\mu)}{3(1+\mu^2)}\right]$.

UNIT – V

CHOOSE THE CORRECT ANSWER:

1.When a uniform string or chain hangs freely between two points not in the same vertical line, the curve in which it hangs under the action of gravity is called __?

a) Catenary

b) Common catenary

c) Directrix catenary

d) None of these

2. The weight per unit length of the chain or string is constant, the catenary is called__

a) Catenary

b) Common catenary

c) Directrix catenary

d) None of these

3. The intrinsic equation of the catenary is

a) $c = s \tan\psi$

b) $s = c \tan\psi$

c) $c = s \cos\psi$

d) None of these

4. The length of the perpendicular from the foot of the ordinate on the tangent at any point of the catenary is

a) Parallel

b) Perpendicular

c) Constant

d) None of these

5. The neighborhood of the lowest point of the catenary the curve approximates in form to a __

a) Circle

b) Square

c) Hyperbola

d) Parabola

6. When a string is very tightly stretched between two points, the shape is nearly__

a) Hyperbola

b) Parabolic

c) Circle

d) Ellipse

7. Any problem in which the sag in small compared with the span, we may assume that the chain hangs in the form of a __?

a) Circle

b) Hyperbola

c) Parabola

d) None of these

8. In a catenary the relation between y and ψ is

a) $s = c\ tan\psi$

b) $y = c\ sec\psi$

c) $c = y\ sec\psi$

d) $c = s\ tan\psi$

9. In a catenary the relation between y and s is

a) $y^2 = c^2 + s^2$

b) $c^2 = y^2 + s^2$

c) $s^2 = c^2 + y^2$

d) None of these

10. In a catenary the relation between s and z is

a) $s = c \cosh \frac{z}{c}$

b) $s = c \tanh \frac{z}{c}$

c) $s = c \sinh \frac{z}{c}$

d) None of these

ANSWERS:

1) a 2) b 3) b 4) c 5) d 6) b 7) c 8) b 9) a 10) c.

TWO MARK QUESTIONS

11. Define Catenary.

12. Define Common catenary.

13. Define Span.

14. Define Sag.

15. Write the intrinsic equation of the catenary.

16. Write the Cartesian equation of the catenary.

17. Define Parameter of the catenary.

18. Define Directrix of the catenary.

19. Define Parabolic catenary.

20. Define Suspension bridge.

FIVE MARK QUESTIONS

21. Show that the shape of a uniform string hanging under gravity is a catenary.

22. Show that the length of a chain whose ends are tied to gether and which in hanging over a circular pulley of radius a so as to be in contact with two-third of the circumference of the pulley is $a\left[\frac{3}{\log(2+\sqrt{3})}+\frac{4\pi}{3}\right]$.

23. A uniform chain of length 2l is to be suspended from two points A and B. In the same horizontal line so that either terminal tension is n times that at the lowest point. Show that the span AB must be $\frac{2l}{\sqrt{n^2-1}}\log\left[n+\sqrt{n^2-1}\right]$.

24. . A uniform chain of length l is to be suspended from two points A and B. In the same horizontal line so that either terminal tension is n times that at the lowest point. Show that the span AB must be $\frac{l}{\sqrt{n^2-1}}\log\left[n+\sqrt{n^2-1}\right]$.

25. A chain of length 2l hangs between two points A and B on the same level. The tension both at A and B is 5 times at the lowest point. Prove that the horizontal distance between A and B is $\frac{l}{\sqrt{6}}\log\left[5+2\sqrt{6}\right]$.

26. A uniform of chain of length l is suspended from two points A and B in the same horizontal line. If the tension both at A and B is two times at the lowest point. Show that AB = $\frac{l}{\sqrt{3}}\log\left[2+2\sqrt{3}\right]$.

27. A chain of length l which can just bear a tension of n times its weight is stretched between two points in the same horizontal line. Show that the least sag in the middle is $l\left[n-\sqrt{n^2-\frac{1}{4}}\right]$.

28. The given length 2l of a uniform chain has to be hung between two points on the same level and tension has exceed the weight of the length k of the chain. Show that the greatest span is $\sqrt{k^2-l^2}\log\left(\frac{k+l}{k-l}\right)$.

29. The rope of length 2l feet is suspended between two points at the same level and the lowest point of the rope is b feet below the points of suspension. Show that the horizontal component tension is $\frac{w}{2b(l^2-b^2)}$. W being the weight of the rope per feet of its length.

30. A heavy uniform string of length l is suspended from a fixed point A and its other end B is rulled horizontally by a force equal to the weight of a length a of the string. Show that the horizontal and vertical distance between A and B are $a\, sinh^{-1}\left(\frac{l}{a}\right)$ and $\sqrt{l^2+a^2}-a$.

TEN MARK QUESTIONS

31. To obtain the Cartesian equation of the common catenary.

32. A kite is flying at a height h with a length l of the string paid out and with the vertex of the catenary on the ground. Show that at the kite the inclination of the string is $2\tan^{-1}\frac{h}{l}$. If w is the weight per unit length of the string. Show that the tension at the ground and at the kite are $\frac{w(l^2-h^2)}{2h}$, $\frac{w(l^2+h^2)}{2h}$.

33. A string hangs with its ends tied at two points α, β are the inclination to the horizon of the tangent at the extremities of a portion of the string and l the length of the portion of the extremities lie on one side of the vertex. Show that the height of one extremity above the other is $\frac{\sin\frac{1}{2}(\alpha+\beta)}{\cos\frac{1}{2}(\alpha-\beta)}$.

34. A string of length 2l hangs over two small smooth pegs in the same horizontal level. Show that if n is the sag in the middle the length of either part of the string that hangs vertically in $h+l-\sqrt{2hl}$.

35. A uniform chain of length 2l is suspended by its which are on the same horizontal level. The distance apart 2a of the ends is such that the lowest point of the chain is at a distance a vertically below the ends. Prove that if 'c' be the distance of the lowest point from directions of the catenary then $\frac{2a^2}{l^2-a^2} = \log\left(\frac{l+a}{l-a}\right)$ and $\tanh\left(\frac{a}{c}\right) = \frac{2al}{l^2+a^2}$.

36. State and prove parabolic catenary.

37. The span of a suspension bridge is 100 meters and the sag at the middle of the chain in 10 meters. If the total load on each chain is 750 quintals. Find the greatest tension in each chain and tension at the lowest point.

38. A uniform chain of length 2l has its end attached to two points in the same horizontal line at a distance 2a apart. If l is only a little greater than a. Show that the tension in the chain is approximately equal to a weight to length $\sqrt{\frac{a^3}{b(l-a)}}$.

39. If a chain of length 2l is suspended from two points A, B on the same level $\frac{l}{n}$. Show that the horizontal span AB is equal to $l\left(n-\frac{1}{n}\right)\log\left(\frac{n+1}{n-1}\right)$.

40. A chain of length 2l has one end tied at B and the other end attached to a small heavy ring A which can slide on a rough horizontal rod which passes through B. If the weight of the ring 'n' times that of the chain and if μ is the coefficient of friction. Show that for the system to be in equilibrium the greatest possible distance 2a of the ring from B is $\frac{2l}{\lambda}\log\left(\lambda+\sqrt{\lambda^2+1}\right)$ where $\frac{1}{\lambda} = \mu(2n+1)$.